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A Discourse on Method

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To Marshall Clagett
A Great ''Great-Grandfather''

ABBREVIATIONS

AT	René Descartes, *Oeuvres de Descartes,* ed. Charles Adam and Paul Tannery, 12 vols. (Paris, 1897–1913).
DA	Alhazen, *De aspectibus,* ed. Friedrich Risner, *Opticae thesaurus* (1572; reprint ed., New York: Johnson Reprint, 1972).
DMS	Roger Bacon, *De multiplicatione specierum,* ed. David C. Lindberg, *Roger Bacon's Philosophy of Nature* (Oxford: Clarendon, 1983).
GW	Johann Kepler, *Gesammelte Werke,* ed. Max Caspar and Franz Hammer, 19 vols. (Munich, 1938–75).
OF	Pierre de Fermat, *Oeuvres de Fermat,* ed. Paul Tannery and Charles Henry, 4 vols. (Paris, 1891–1912).
PC	John Pecham, *Perspectiva communis,* ed. David C. Lindberg, *John Pecham and the Science of Optics* (Madison: University of Wisconsin Press, 1970).
TL	Christiaan Huygens, *Traité de la lumière* (1690; reprint ed., Brussels: Impressions Anastaltique, Culture, et Civilisation, 1967).
WP	Witelo, *Perspectiva,* ed. Friedrich Risner, *Opticae thesaurus* (1572; reprint ed., New York: Johnson Reprint, 1972).

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PREFACE

Now, whoever does not exist cannot be deceived, and so I exist if I am deceived. Therefore, since I exist if I am deceived, how can I be deceived about my existing when it is certain that I exist if I am deceived?

Most of us who are at least passingly familiar with Descartes's philosophical writings would associate this passage with Descartes's "cogito" argument and would ascribe it unhesitatingly to the *Meditations.* Some of us might even pinpoint it to the beginning of the second "Meditation," where Descartes entertains the possibility of a God whose sole purpose is to mislead us at every turning. We would be wrong. The quotation comes not from the *Meditations,* but from St. Augustine's *De civitate Dei* (XI, 26).

My intent here is not to play up the historical significance of this particular connection between Descartes and Augustine. It is, rather, to underscore its present obscurity as an example of the lack of historical awareness in a good deal of modern scholarship about Descartes.[1] That so few of us are alive to the connection, despite its obviousness, indicates just how thoroughly Descartes has been wrenched from his own past and transplanted into our present by scholars bent on evaluating his ideas according to modern philosophical and scientific canons. As a consequence, the historical roots of Cartesian thought have been all but severed over the past several decades in the effort to understand that thought through its contemporary, and therefore "relevant," ramifications.

Above all, I hope in this monograph to show how sterile such an exclusively "presentist" approach can be and, by contrast, how fruitful an historical understanding of Descartes's thought really is. I have chosen to focus upon his thought about light and refraction for two reasons. First, Descartes was convinced that his ideas on light were central to his more general philosophy of nature. Second, those ideas contain a variety of puzzling features which, despite numerous attempts to grapple with them on metaphysical, mathematical, and methodological grounds, have never been adequately explained. The reason for this failure, I maintain, is that no one has yet traced those ideas with sufficient care back to the medieval optical tradition. Therefore, my task in the following essay is to demonstrate, first, that Descartes's theory of light and refraction was indeed rooted in this tradition and, second, that we can make the best sense of this theory by evaluating it within that historical context.

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[1] Bernard Williams, for instance, relegates the connection between Descartes and Augustine to several asides in his influential *Descartes: The Project of Pure Enquiry* (Harmondsworth: Penguin, 1978). Margaret Wilson, in her equally influential study of the *Meditations* (*Descartes* [London: Routledge and Kegan Paul, 1978]), refers to it just once, and then only obliquely. In all fairness, I should point out that by no means all modern students of Descartes have ignored or downplayed his debt to previous thinkers. One of the earliest and most important scholars to examine and expose this debt in detail was the famous medievalist Étienne Gilson, whose studies on Descartes culminated with the *Études sur le rôle de la pensée médiéval dans la formation du système cartésien* (Paris: Vrin, 1930). For a recent example of historical sensitivity to the links between Cartesian and Scholastic thought, see Marjorie Grene, *Descartes* (Minneapolis: University of Minnesota Press, 1985). An intriguing post-Augustinian parallel to Descartes's "cogito" argument—at least as far as establishing the grounds of doubt are concerned—crops up in the preface to Francisco Sanches's *Quod nihil scitur* of 1581, in Francisco Sanches, *Opera Philosophica,* ed. Joaquim de Carvalho (Coimbra, 1955); see pp. 1–4, esp. p. 2, ll. 3–12 (my thanks to Joseph Dauben for alerting me to this work).

To acknowledge every institutional and personal contribution to this work would, of course, be impractical, if not impossible, so the following list is necessarily truncated. For a start, I wish to acknowledge generous financial support from the regents of the University of California, not only through intramural research funding but also through a Regents' Junior Faculty Fellowship for 1984–85. Likewise, I owe thanks to the National Endowment for the Humanities for a 1984 Summer Institute stipend that provided the occasion to be in Washington, D.C., where I made use of the resources of Dumbarton Oaks, the Folger Shakespeare Library, and the Library of Congress. Grateful acknowledgement is also due to the Laboratory for Historical Research at the University of California, Riverside, whose excellent computer facilities have greatly eased the burden of writing for me. I am much indebted both to its director, Charles Wetherell, and to his assistant, Mariel Davis, for their technical advice and support. On a more personal level, I want particularly to thank David Lindberg of the University of Wisconsin for his editorial help and encouragement. I only wish I could fully repay his generosity. To Marshall Clagett of the Institute for Advanced Study I owe equal gratitude for the same reasons. Finally, for helping me express myself more clearly and effectively than I might otherwise have done, I thank Charles Wetherell and Naomi Liebler, both of whom wield a red pen with penetrating common sense.

I. INTRODUCTION

Descartes was doubly gifted with a rare precision of mind and an equally unusual precision of expression. So patently do both come through in his writings that it is difficult not to mistake his lucidity for originality. We see in his ideas such a striking clarity and simplicity that it is almost as if we were watching pure thought in the making and, out of that, the construction of clean, new edifices of reason. Of course he wanted us to see it that way. He was eager to impress upon us the novelty of his own ideas, often by contrasting them with the obfuscations of the "Schoolmen." And his expressions of contempt for those same Schoolmen and their dogma only serve to heighten our awareness of the gulf between his new, progressive ideas and their old, reactionary ones.[1]

Whether or not Descartes was genuinely convinced of the novelty of his thought is beside the point. He was genuinely convincing. That is why, even up to recent times, he has been so consistently portrayed as a philosophical and scientific pioneer:

> By an almost universal agreement among philosophers and historians, René Descartes is considered the originator of modern philosophy, or at least the first important philosopher of our times. If we add to this the common belief that philosophy points the way for developments in other fields, it will be evident that to Descartes is ascribed an importance comparable to that of the beginnings of intellectual culture in Greece or of the origin and spread of Christianity in the Mediterranean regions and surpassing all other events in history.[2]

"For Descartes, nothing less would suffice," says Allen Debus in the same vein, "than to discard completely all past knowledge and to begin anew, accepting as axiomatic only God and the reality of one's own existence (*Cogito, ergo sum*)." But that is not all. "From this foundation," Debus continues, "Descartes was prepared to deduce the entire universe and its laws."[3]

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[1] Descartes's many references to "l'École" and the "Philosophes" ("ou plutost les Sophistes," as he reminds us in *Le Monde*), which were pointed at Aristotle and his Scholastic followers, are invariably critical of their complex formal and terminological distinctions.

[2] Laurence J. Lafleur, "Descartes' Place in History," in *Cartesian Essays: A Collection of Critical Studies*, ed. Bernd Magnus and James Wilbur (The Hague: Martinus Nijhoff, 1969), 3.

[3] *Man and Nature in the Renaissance* (Cambridge: Cambridge University Press, 1978), 106. This view reflects an acceptance of Descartes's own claim in the *Meditations* that he really did reject all previously acquired philosophical knowledge and opinion as a starting point for finding absolutely clear and distinct ideas upon which to build a new and consistent system of knowledge. See Richard H. Popkin, *The History of Skepticism from Erasmus to Descartes* (New York: Harper, 1968), 132–96, for an account of Descartes's thought as an outgrowth of "constructive or mitigated scepticism."

Free from the dogmatic constraints of Scholasticism and secure in the originality of his own thought, Descartes owed little or nothing to the Scholastic tradition beyond the opportunity to lift himself above it.

Many intellectual historians have accepted this assessment, seeing in Descartes's work a genuinely new effort to rebuild natural philosophy, or science, from the ground up.[4] Indeed, as more than one historian characterizes it, that effort was nothing less than a mission—one that could only be fulfilled on the basis of an entirely new method of philosophical and scientific inquiry consistent with the canons of perfect clarity and distinctness.[5] Scholastic natural philosophy has thus assumed the role of antithesis or foil to this mission, so that the relationship between the two has usually been taken as negative rather than positive. As a result, historians of philosophy and science have often tended to treat Descartes's thought as more or less self-generated and self-contained.

A good case in point—and indeed the case in point for this essay—is his theory of light-refraction in the *Dioptrique.* As we shall see in due course, this theory and its attendant proof of the sine-law present a number of vexing logical inconsistencies which have so far not found adequate explanation. The reason, as we shall also see, is quite simple. With very few exceptions, both the theory and the proof have been treated in isolation from previous Scholastic theories. Why? Presumably because both the theory and the proof sprang from Descartes's single-minded attempt to rebuild philosophy and science upon his new methodological foundations.

But Descartes's work in optics, as in everything else, marks a culmination as well as a beginning. Common sense alone tells us that it could not have been a *creatio ex nihilo.* Common sense likewise tells us that the "something" out in which it *was* created must have been proximate. The solution is therefore as simple as the problem: to reevaluate Descartes's theory of refraction as a logical outgrowth of, rather than a radical departure from, Scholastic theory and thereby make sense of his optical method and its shortcomings.

The reevaluation itself will unfold in six stages marked by separate sections. In the first of these (Chapter II), I examine certain historical and historiographical issues that have so far colored our understanding of Descartes's optical work. We will see that these issues tend to center on particular assumptions about both his method and his sources. Having established a proper historical context, I then turn in Chapter III to Descartes's basic theory of light in order to explain how it provides the foundation for his proof of the sine-law and why it seems logically inconsistent with that proof in crucial ways. In the fourth chapter, which focuses upon Scholastic light-theory, I discuss the suppositional grounds of that theory and then look at precisely how those grounds could have served as a logical springboard for Descartes's theory. In Chapter V the Scholastic analysis of refraction is examined, my object being not only to show how it fits within the general framework of medieval light-theory, but also to assess its underpinnings in specific critical assumptions.

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[4] As Desmond Clarke puts it, "the almost canonical approach [among modern scholars] has been to adopt Descartes' philosophy as a guide to what he thought he was doing in science; perhaps more narrowly, to read the *Meditations* as Descartes' contribution to philosophical literature and to infer much of what he must have meant, or should have said if he were consistent, about scientific method from his first philosophy": *Descartes' Philosophy of Science* (Manchester: Manchester University Press, 1982), 2; see pp. 7–12 for a summary of the various ways in which Descartes's grand methodological project can be, and has been, interpreted. See also Wilson, *Descartes,* esp. chap. 1.

[5] See A. Boyce Gibson, *The Philosophy of Descartes* (New York: Russell and Russell, 1967), chap. 2. See also John A. Schuster, "Descartes and the Scientific Revolution: 1618–1644, An Interpretation" (Ph.D. diss., Princeton University, 1977), and "Descartes' *mathesis universalis:* 1619–28," in *Descartes: Philosophy, Mathematics and Physics,* ed. Stephen Gaukroger (New York: Barnes and Noble, 1980), 41–96.

The sixth and penultimate section is the heart of my proposed reevaluation. Here we see how readily Descartes's proof of the sine-law follows from pursuing certain implications of the Scholastic refraction-analysis to their logical conclusion, and that the very inconsistencies of his analysis and proof outlined in Chapter III are inevitable if the line of logic indicated by those implications is actually followed. In short, Descartes's proof actually does make sense when its rationale is traced back to the proper source. Finally, in Chapter VII, I complete the reevaluation by explaining how, tangled though it was in Scholastic roots, Descartes's misguided resolution of the problem of refraction was nonetheless crucial in preparing the field of theoretical optics for the phenomenal growth it underwent in the second half of the seventeenth century.

II. THE BASIC ISSUES

An Historical Overview

It is a common misconception that, by appending it to his *Discours de la Methode* of 1637, Descartes meant to offer the *Dioptrique* as an illustration of how to apply the method outlined in the prefatory *Discours.*[1] That such was not his purpose is evident from a letter of 1638, in which he explained that he meant only to give a sample of the kinds of results his method could produce. He could not, he added by way of apology, "demonstrate the [actual] use of that method . . . because it prescribes an order of investigating things that is quite different from the order in which I believed they ought to be explained."[2] Still, the implication is clear. The results contained in the *Dioptrique* were arrived at, or discovered, by means of his new anti-Scholastic method.

Among those results, certainly the most historically significant was the putative discovery of the sine-law of refraction.[3] I say "putative" because

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[1] The *Discours de la Methode* was published along with three appended treatises: the *Dioptrique,* the *Météores,* and the *Géométrie,* in that order. The very structure of the publication as a whole suggests that the three appended treatises were intended to serve as illustrative cases of the method; see, for example, Leon Roth, "The *Discourse of Method,*" *Mind* 46, 181 (1937): 32–43, and L. J. Beck, *The Method of Descartes: A Study of the Regulae* (Oxford: Clarendon Press, 1952), 176–89 and 210–14. In reality, the arrangement was gratuitous; Descartes happened to have all four tracts ready to hand and decided to publish them all together at the same time. See A. I. Sabra, *Theories of Light from Descartes to Newton* (London: Oldbourne, 1967), 60, n. 48, for a chronological summary of the entire tract's composition; see also J. F. Scott, *The Scientific Work of René Descartes (1596–1650)* (London: Taylor and Francis, 1952), 8–10. The *Dioptrique* itself was planned and executed long before its actual publication. In fact, we have a clear indication from a letter to Mersenne in late November, 1630, that by then the *Dioptrique* was already well underway: "Ie croy que ie vous enuoyeray ce discours [i.e., *Le Monde*] auant que de vous enuoyer le reste de la Dioptrique" (*AT,* 1: 179).

[2] Descartes to Vatier, 22 February 1638: "mon dessein n'a point esté d'enseigner toute ma Methode dans le discours où ie la propose, mais seulement d'en dire assez pour faire iuger que les nouuelles opinions, qui se verroient dans la Dioptrique et dans les Meteores, n'estoient point conceuës à la legère, et qu'elles valoient peutestre la peine d'estre examinées. Ie n'ay pû aussi monstrer l'vsage de cette methode dans les trois traittez que i'ay donnez, à cause q'elle prescrit vn ordre pour chercher les choses qui est assez different de celuy dont i'ay crû deuoir vser pour les expliquer" (*AT,* 1: 559). Unless otherwise noted, all English translations in this essay are mine.

[3] Briefly, the sine-law states that when light passes obliquely through the interface of two transparent media of different optical densities (e.g., from air to water) it will be bent or refracted in such a way that the sine of the angle of incidence will be constantly proportional to the sine of the angle of refraction. Thus, in Figure 1, the ray of light DC strikes interface ACB between two different transparent media at angle of incidence DCF (signified by i) and refracts along EC at angle ECG (signified by r), the two angles being taken in respect to the "normal" FG. According to the sine-law, then, no matter what the value of i (up to a certain point when the light reflects), sine i:sine r = constant.

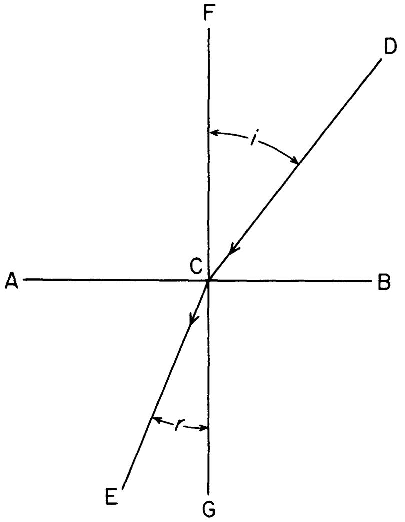

FIGURE 1

it is by now well established that, although it made its first *published* appearance in the *Dioptrique,* the sine-law had been known in various forms to several optical researchers long before that.[4] So whatever claim Descartes might have to priority of actual discovery is doubtful at best. Indeed, we need only look at a standard physics text to see that history has denied him such priority; "Snell's Law" it is and will probably remain forever.

While Descartes may not have been the first to find or formulate the sine-law, he *was* the first to offer a complete, systematic justification or proof of that law. Equally important, his proof was based on what he considered to be a radically new theory of light-radiation, centered on the idea that, instead of being a qualitative change in transparent media, light consists of mechanical impulses through those media.[5] The beauty of this new theory, as he saw it, was that it simplified the science of optics by allowing all optical phenomena (refraction in particular) to be treated in terms of mechanical interactions or physical collisions. In short, Descartes was sure he had reduced optics to a special case of the science of motion. The sine-law was therefore not simply an empirical fact or discovery for him. It was a logical and necessary consequence of his new physics of light. That is doubtless why he published the *Dioptrique*—in the hope that his derivation and proof of the sine-law would be warmly received as an earnest of the power both of his new light-theory and of his new method.

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[4] See Appendix A for a brief summary of the pre-Cartesian history of the sine-law.

[5] Descartes's general theory of light and the subsequent derivation of the sine-law occupy only the first two "Discourses" of the *Dioptrique* (*AT* 6: 81–105). The remaining eight "Discourses"—which will not concern us here—deal with sight and ways to improve it with lenses. In the fourth chapter of this essay we discuss at length the theory that light is a qualitative change in transparent media. Suffice it for now to say that it was Scholastic, and that Descartes's theory was intended to supplant it.

Philosophy, Newton once observed, is a litigious lady, and she certainly was for Descartes. No sooner had he published the *Dioptrique* than the account of refraction in it came under attack from a variety of critics. Of these, Pierre de Fermat was without question the most effective and persistent. His objections to Descartes's proof were doubly motivated. First, Fermat doubted the validity of the sine-law itself and apparently remained dubious until 1662 when, ironically enough, he demonstrated the law to his own satisfaction.[6] Second, Fermat was suspicious of Descartes's procedure, because he believed that Descartes had "accommodated his means [of proof] to his conclusion."[7] Whether Fermat's suspicion was fueled by distrust of the law, or vice-versa, is unclear. What is clear is that for over a quarter of a century he relentlessly pressed his attack upon Descartes's method, undermining it bit by bit. And as he clarified his objections, Fermat managed to bring into ever sharper focus certain logical inconsistencies in the Cartesian account. Things finally came to a head in the early 1660s when, having perfected his own counter-proof, Fermat had the means to put Descartes's demonstration to rest once and for all. The logical superiority of his counter-proof posed such a sharp contrast to the inadequacy of Descartes's alternative that only Descartes's most ardent supporters could fail to concede that his line of reasoning was fallacious.[8]

Such a concession leads inevitably to the question of how Descartes could conceivably have discovered the right law by following such an erroneous line of reasoning.[9] Only two explanations seem possible: either he was phenomenally lucky, or he did not actually discover the law. Since the first of these is no explanation at all (luck or chance being by definition irrational), we can dismiss it for the moment. That leaves no apparent choice but to admit that Descartes did not actually discover the law. This is tantamount to saying that he knew the law beforehand and subsequently devised a spurious proof to vindicate it. In fact, it has commonly been suggested that he plagiarized the law from Snel (or Snell), which would certainly explain why, as an *ad hoc* means to an *a priori* conclusion, Descartes's proof is so riddled with inconsistencies.[10] We are thus faced with a dilemma. On the one hand, we can accept that he actually discovered the sine-law independently, in which case we admit that we cannot make sense of the discovery because the method by which he achieved it was arbitrary and unreasonable. On the other hand, we can accept that he did not really discover the law, in which case we admit that he stole it and offered a trumped-up rationalization in the hope of passing the law off as his own.

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[6] Fermat was certainly doubtful as of 10 March 1658, when he wrote to Claude Clerselier that, as far as he was concerned, "la véritable raison ou proportion des réfractions est encore inconnue" (*OF*, 2: 374).

[7] "l'Autheur a accomodé son *medium* a sa conclusion" (*AT*, 1: 464 [ll. 21–22]).

[8] See Appendix B for details on Fermat's controversy with Descartes and the Cartesians.

[9] "Where Descartes got the law, or how he got it," laments Michael Mahoney, "remains a mystery" (*Dictionary of Scientific Biography* [New York: Scribners, 1971], 4: 59b–60a). See also Ernst Mach, *The Principles of Physical Optics*, trans. J. S. Anderson and A. F. A. Young (New York: Dover, 1953), 32–34.

[10] Isaac Voss was the first to level the charge of plagiarism at Descartes in his *De lucis natura et proprietate* of 1662, chap. 16, p. 26, and his *Responsio ad objecta Joh. De Bruyn* of 1663 (in the same volume, pp. 32–33). It was echoed later by, among others, Gottfried Leibniz in his *Discourse on Metaphysics*, trans. Peter Lucas and Leslie Grint (Manchester: Manchester University Press, 1953), art. 22, p. 40; Joseph Priestly in *The History and the Present State of Discoveries Relating to Vision, Light, and Colour* (London, 1772), 1: 102; and Mach, *Principles*, 33. The factual basis for this charge is non-existent, and the actual grounds for the case are

Actually, there is a relatively easy solution to this dilemma. Perhaps Descartes's method of proof in the *Dioptrique* was not really arbitrary or unreasonable. Perhaps he was following a real train of logic—not the "right" train to be sure, but a train nonetheless—and perhaps this train led directly to the sine-law. If so, then he need not have known the law beforehand, in which case he need not have plagiarized it. The resolution of the dilemma thus lies in finding a clear rational order behind the ostensibly incoherent procedure Descartes used to establish the sine-law. That, in brief, is the purpose of this essay.[11] A. I. Sabra and David Lindberg have already paved the way by connecting Descartes to the medieval optical tradition through certain key Perspectivist ideas that crop up in the *Dioptrique.*[12] Thanks primarily to their efforts, we know that Descartes was indebted to the Perspectivists for particular elements of his refraction-analysis. In what follows, however, I intend to show that Descartes's debt to the Perspectivists was even more profound than that. Not only does his account contain specific Perspectivist ideas; it is in fact *systematically* dependent upon the Perspectivist theory of refraction. The real method behind Descartes's method of refraction-analysis thus has its basis in that theory. And it was this underlying method that opened a path of specious reasoning which, paradoxically, led Descartes straight to the sine-law.

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so conjectural as to be thoroughly unconvincing (see Schuster, "Descartes and the Scientific Revolution," 360, n. 89). It is, of course, possible that Descartes arrived at the sine-law through straightforward empirical discovery. To my knowledge, however, no one has forwarded such a claim seriously, nor did Descartes himself make it. Moreover, the likelihood of Descartes's stumbling upon the sine-law by experiment alone is exceedingly slim, especially given the naked-eye conditions under which such experimentation would have been conducted. For a detailed argument against the likelihood of experimental "discovery" of the sine-law, see my "Ptolemy's Search for a Law of Refraction: A Case-Study in the Classical Methodology of 'Saving the Appearances' and its Limitations," *Archive for History of Exact Sciences* 26 (1982): 221–40.

[11] The best attempt so far at reconstructing Descartes's path to the sine-law can be found in Schuster's "Descartes and the Scientific Revolution," 268–368. In sharp contrast to the thorough historical scholarship of this account, Brendan P. Minogue's "Cartesian Optics: A Test Case for the Partial Interpretationist's Account of Models" (Ph.D. diss., Ohio State University, 1974) offers a rather freewheeling and abstract philosophical reconstruction. The best I can say of this attempt is that it betrays an appalling insensitivity to the actual context, both historical and intellectual, within which Descartes's reasoning developed.

[12] See A. I. Sabra, *Theories of Light,* 17–158, and David C. Lindberg, "The Cause of Refraction in Medieval Optics," *The British Journal for the History of Science* 4 (1969): 23–38. I am particularly indebted to these works and their authors for the insights they have provided about Descartes's connection with the Perspectivists.

Descartes's Perspectivist Sources

How closely we can link Descartes and the Perspectivists depends, first, on knowing how and how well he actually knew the Perspectivist sources. Unfortunately, we cannot be as certain on this head as we might like, because Descartes was habitually sparing of direct citations in his published works and only somewhat less so in his correspondence. Why he was so is an open question, but there is ample reason to suppose that his knowledge of Scholastic sources was far less limited than his silence indicates.[13] Perhaps it would be more accurate to say that his knowledge of Scholastic *ideas* was less limited than his silence indicates. We know, for example, that his formal education at the Jesuit Collège de la Flèche and the University of Poitiers introduced him to Scholastic writings, even if only at second hand, and we have a variety of clues in his own writings to suggest a fairly good grasp of the Scholastic philosophical tradition.[14] Likewise with the Perspectivist optical tradition, the avenues for his knowledge were as open as the availability of direct or indirect sources would allow.

In its narrowest sense, the "Perspectivist tradition" comprises a select group of optical works that took their original inspiration from Alhazen's *Kitāb al-manāẓir,* which was written in Arabic during the early eleventh century and translated into Latin as *De aspectibus* sometime in the early thirteenth.[15] Aside from this seminal work, the tradition includes Roger Bacon's *Perspectiva* (1267?) and *De multiplicatione specierum* (early 1260s?), Witelo's *Perspectiva* (ca. 1275), and John Pecham's *Perspectiva communis* (ca. 1279).[16] These treatises were enormously influential during the succeeding three centuries because all except Bacon's two works entered the university curriculum as texts for the teaching of mathematics and optics. For instance, an Oxford statute of the late sixteenth century mandated the use of either Euclid's *Elements* or Witelo's *Perspectiva* for the teaching of geometry.[17] The Perspectivist tracts also served as bases for numerous Scholastic commentaries on vision and light theory. Among these, several sets of *questiones* on the science of *perspectiva* still survive in manuscript form, as do a number of specialized works dealing with the rainbow and parabolic mirrors.[18] In addition, a good deal of Perspectivist lore appears not only in later medieval commentaries on Aristotle's *De anima* and *Meteorology*, but in commentaries on the *Sentences* as well.[19]

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[13] Although it might be taken as an indication of Descartes's mean-spiritedness, I suspect that the relative paucity of direct citations in his works—especially of Scholastic authorities—is due less to an ungenerous nature than to a desire not to seem to be arguing from authority.

[14] See, e.g., Étienne Gilson, *Index scolastico-cartésien* (1912; reprint ed., New York: Franklin, 1964) and *Études sur le rôle.*

[15] A critical Arabic edition, with English translation and commentary, of Ibn al-Haytham's (Alhazen's) *Kitāb al-manāẓir* is at present being completed by A. I. Sabra. No critical edition of the Latin version is yet available. In tying the Perspectivist tradition directly to Alhazen's *De aspectibus,* I go against usual historical practice by excluding Robert Grosseteste from the Perspectivist ranks, since it is fairly certain that he was unaware of Alhazen's work. This omission is of little real consequence, however, since Grosseteste's work was perpetuated and expanded by his putative disciple, Roger Bacon, whom I do include. See A. C. Crombie, *Robert Grosseteste and the Origins of Experimental Science, 1100–1700* (Oxford: Clarendon Press, 1953), for a more positive evaluation of Grosseteste's role in the development of Perspectivist optics.

[16] A critical Latin edition of Bacon's *Perspectiva* (part 5 of the *Opus maius*) can be found in *The 'Opus Majus' of Roger Bacon,* ed. John H. Bridges, 2 vols. (London: Williams and Norgate, 1900), vol. 2; henceforth all citations will be from that edition. There is also an English translation to be found in *The 'Opus Majus' of Roger Bacon,* trans. Robert B. Burke, 2 vols. (Philadelphia: University of Pennsylvania Press, 1928), vol. 2. David C. Lindberg's critical editions of the *De multiplicatione specierum* and the *Perspectiva communis* also contain English translations. As yet, there is no complete modern critical edition of Witelo's *Perspectiva,* although books 1 and 5 have recently been edited and translated into English: for book 1, see Sabetai Unguru, *Witelonis Perspectrivae liber primus,* Studia Copernicana 15 (Warsaw: Polish Academy of Sciences, 1977); for book 5, see A. Mark Smith, *Witelonis Perspectivae liber quintus,* Studia Copernicana 23 (Warsaw: Polish Academy of Sciences, 1983).

Just how influential the basic Perspectivist tracts were during the period between roughly 1300 and 1600 can be gauged by the number of manuscripts of each that survive today: twenty of Alhazen's *De aspectibus* (latest, sixteenth century); thirty-nine and twenty-four respectively of Bacon's *Perspectiva* (latest, sixteenth century) and *De multiplicatione specierum* (latest, seventeenth century); twenty-six of Witelo's *Perspectiva* (latest, sixteenth century); and sixty-four of Pecham's *Perspectiva communis* (latest, sixteenth century). Another gauge is the printing history of each up to the early seventeenth century: one edition for Alhazen's *De aspectibus* (1572); one for Bacon's *Perspectiva* (1614); three for Witelo's *Perspectiva* (1535, 1551, 1572); and ten for Pecham's *Perspectiva communis* (1482/83?, 1503, 1504 [Venice], 1504 [Leipzig], 1510, 1542, 1556, 1580, 1592, 1627).[20] Numbers do not tell the whole story, though. Of all these printings, the exquisite tandem edition of Alhazen's and Witelo's treatises, published in 1572 by Friedrich Risner under the title *Opticae thesaurus*, was by far the most influential, primarily because of the extraordinary care and scholarship with which it was compiled.[21]

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[17] See David C. Lindberg's introduction to the Johnson Reprint edition of Friedrich Risner's *Opticae thesaurus*, xxi–xxv, for a discussion of this point (see list of abbreviations for publication data). While Bacon's optical treatises may not have entered the Scholastic curriculum as standard texts, they were nonetheless widely read and influential throughout the later Middle Ages and Renaissance.

[18] For a list of authors and "question-sets" on *perspectiva* in the later Middle Ages, see David C. Lindberg, *A Catalogue of Medieval and Renaissance Optical Manuscripts* (Toronto: Pontifical Institute of Mediaeval Studies Press, 1975). There, too, can be found lists of works dealing with mirrors in general, as well as parabolic or "burning" mirrors, and with the rainbow. For specialized studies of these topics, see William A. Wallace, O. P., *The Scientific Methodology of Theodoric of Freiberg* (Fribourg: Univeristy of Fribourg Press, 1959), and Carl B. Boyer, *The Rainbow: From Myth to Mathematics* (New York: Thomas Yoseloff, 1959).

[19] See, e.g., Peter Marshall, "Nicholas Oresme's *Questiones super libros Aristotelis De Anima*" (Ph.D. diss., Cornell University 1980); Stephen C. McCluskey, "Nicole Oresme on Light, Color, and the Rainbow: An Edition and Translation, with Introduction and Critical Notes, of Book Three of His *Questiones super quattuor libros Meteorum*" (Ph.D. diss., University of Wisconsin, 1974); and Katherine H. Tachau, "The Problem of the *Species in Medio* at Oxford in the Generation after Ockham," *Mediaeval Studies* 64 (1982): 394–443.

[20] These statistics have been drawn from Lindberg, *Catalogue.* An updated account of the manuscript-tradition for Witelo's *Perspectiva* can be found in my *Witelonis Perspectivae liber quintus*, 73–76. The obvious popularity of Pecham's work is due to the fact that it was written as an introductory survey of optics and served as such throughout the Middle Ages and Renaissance (see *PC*, preface, p. 1).

By the time Descartes undertook his optical researches, then, he had available to him in printed form the entire Perspectivist *corpus,* with the exception of the *De multiplicatione specierum.* Meanwhile, a number of fairly influential and popular derivative works dealing specifically with refraction and its various aspects had appeared in print over the sixteenth and early seventeenth centuries. One of the most popular of these was Giambattista della Porta's *Magiae naturalis libri XX,* which included an important discussion of lenses. First published in 1589, this treatise reappeared in several Latin editions (as well as a number of vernacular translations) during the next several years.[22] Less successful, but more scholarly, was his *De refractione,* printed in 1593.[23] Yet another example was Francesco Maurolyco's *Photismi de lumine,* which contained both a general mathematical analysis of refraction and a specific explanation of the rainbow. Although finished by 1567 at the latest, this book did not see publication until 1611.[24] Such works served to perpetuate and extend the Perspectivist optical tradition, as well as to stimulate interest in various aspects of refraction.

But in every respect, these works were overshadowed by Kepler's *Ad Vitellionem paralipomena* ("Supplement to Witelo['s *Perspectiva*]") of 1604.[25] As the title suggests, Kepler's intent was to offer various emendations and additions to Witelo's original work. Specifically, he attempted to find an exact measure of atmospheric refraction in order, finally, to perfect observational accuracy in astronomy. That measure eluded him, but during the search he undertook a thorough and critical evaluation of the Perspectivist account of refraction with specific reference to Witelo and Alhazen. Short of using the actual sources themselves, one could hardly find a more effective way to learn the Perspectivist theory of refraction than through the synopsis that Kepler provides on the basis of this evaluation.[26]

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[21] Risner's edition was collated from several manuscripts and includes a variety of useful cross-references between Alhazen and Witelo, as well as between them and earlier optical writers such as Euclid and Ptolemy. Risner himself wrote a brief compendium on optics, *Opticae libri quatuor,* which was published posthumously in 1606.

[22] The *Magiae naturalis libri XX* of 1589 was a greatly expanded version of a work first published in 1558 as *Magiae naturalis libri IV.* Unlike the earlier version, the 1589 edition contained a book (the seventeenth) devoted to lenses and refraction; for a good summary of Porta's thought on refraction and lenses, see David C. Lindberg, "Optics in Sixteenth-Century Italy," *Annali dell'Istituto e Museo di Storia della Scienza,* fasc. 2, (1983): 142–48. Within less than 80 years of its first publication, this work saw at least seven Latin editions (1591, 1607, 1619, 1644, 1651, 1664); an Italian translation (1611), the seventeenth book of which has been republished in modern form by Vasco Ronchi, ed., *Scritti di ottica* (Milan: Polifilo, 1968), 151–95; a French translation (1631), republished in modern form (Paris: Daragon, 1910); and an English translation (1658, 1669), reprinted in facsimile (New York: Basic Books, 1957).

[23] The Latin text, with facing Italian translation, of the *De refractione* can be found in Ronchi, *Scritti,* 196–243. In the very last years of his life, Porta also wrote a manuscript entitled *De telescopio,* which, despite the author's intentions, never saw publication.

[24] Evidently written in several stages, starting perhaps as early as 1521, the *Photismi de lumine* was gathered together in its final form by 1567. For part of the Latin edition of 1611, with facing Italian translation, see Ronchi, *Scritti,* 77–131. For an English translation of that same edition, see Henry Crew, trans., *The* Photismi de lumine *of Maurolycus: A Chapter in Late Medieval Optics* (New York: Macmillan, 1940). See Lindberg, "Optics in Sixteenth-Century Italy," 138–41, for a brief account of Maurolyco's theory of refraction.

[25] *Ad Vitellionem paralipomena, quibus astronomiae pars optica traditur* (Frankfurt, 1604), in *GW,* 2; an excellent French translation of this work can be found in Catherine Chevalley, *Johan Kepler. Les fondements de l'optique moderne: Paralipomènes à Vitellion (1604)* (Paris: Vrin, 1982). Unlike the far shorter *Dioptrice* of 1611, which is a straightforward compendium of geometrical optics, the *Paralipomena* is devoted to a complete reconsideration of all aspects of light-theory from simple radiation to refraction.

Although by no means exhaustive, the foregoing account indicates that a wealth of primary and secondary sources was readily available to Descartes in both manuscript and printed editions when he turned seriously to optical research in the 1620s.[27] That he read at least some of these is hardly in question. The real question is which ones he read and in what form he read them. To answer that, we must narrow his options to a reasonable minimum by arranging them in order of likelihood. For instance, we can assume that Descartes would have favored printed over manuscript sources. Among printed sources, moreover, he would have been more likely than not to use readily available ones. And among those, he would have been most likely to use works written by authors whose names are cited in his published works or correspondence. On the basis of these criteria, we can arrange a list of choices in descending order of probability from almost definite to possible:

1. *Almost definite:* In this category we can safely place Witelo's *Perspectiva* and Kepler's *Paralipomena,* since Descartes refers to both authors several times by name in his correspondence and occasional writings (in one particular citation of Kepler, he could only have been referring to him as author of the *Paralipomena*). Which edition of Witelo's *Perspectiva* he used is uncertain, but it was probably Risner's *Opticae thesaurus* of 1572, since that was the most authoritative and accessible of the lot.[28]

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[26] The first chapter of the *Paralipomena* provides an excellent summary of the general Perspectivist theory of light (with which Kepler was in substantial agreement), whereas the first three sections of the fourth chapter provide an epitome of pre-Keplerian theories of refraction (with which he was in substantial disagreement).

[27] See Appendix C for a discussion of the immediate context of Descartes's optical researches.

[28] In addition to a couple of direct references to Witelo in his correspondence (see, e.g., *AT,* 1: 239 and 2: 142), there is in a short scrap attributed to him a complete tabulation of angles of incidence and refraction copied directly from *WP,* book 10, proposition 8, p. 412 (*AT,* 11: 645–646). Aside from at least two mentions of Kepler by name (*AT,* 1: 331, 593), the *locus classicus* for connecting Descartes with Kepler is in a letter to Mersenne of 31 March 1638, where Descartes refers to Kepler as "mon 1er maistre en Optique" (*AT,* 2: 86). According to Chevalley (*Les fondements,* 41–42), the content and context of this quotation make it almost indisputable that the "Kepler" of this passage was the author of the *Paralipomena,* rather than of the *Dioptrice,* with which Descartes was also familiar. Moreover, according to her, it is a virtual certainty that Descartes had already read Witelo's *Perspectiva* at first hand before dealing with either of Kepler's works (see pp. 42–44). The principal evidence against Descartes's use of Risner's compendium is his reference to Witelo according to the form "Vitellio" (which appears in Kepler's *Paralipomena* and in the 1535 and 1551 editions of the *Perspectiva*) rather than to Risner's form "Vitello." As Sabra (*Theories of Light,* 72, n. 13) points out, however, this is anything but conclusive evidence that Descartes was unfamiliar with the Risner edition.

2. *Probable:* Alhazen's *De aspectibus* is the only one to fall into this category. Given our certainty that Descartes knew Witelo's work through Risner's *Opticae thesaurus,* we have every reason to suppose he read Alhazen's tract in the same edition. After all, it is difficult to believe that at some time in the course of his optical researches Descartes would not have felt compelled to read the *De aspectibus,* if only because of its scholarly reputation and ready availability at the time.
3. *Possible:* For the sake of convenience, we can lump the rest of the printed sources, including Pecham's *Perspectiva communis* and Bacon's *Perspectiva,* under this category. We have no compelling reason either to believe or not to believe that he read any of these, and although he mentions Roger Bacon once by name in his correspondence, it is only to deny the charge that he borrowed his theory of refraction from him.[29] The only safe conclusion to be drawn from this is that whoever leveled the charge at Descartes must have been familiar with one or another of Bacon's works.

In summary, then, we can be reasonably certain that Descartes had access to, and drew from, at least three basic Perspectivist sources in his optical researches: Alhazen's *De aspectibus,* Witelo's *Perspectiva,* and Kepler's *Paralipomena.* Less certain, but still quite possible, is that he was familiar with Pecham's *Perspectiva communis* and Bacon's *Perspectiva* through the printed versions that were available by the early seventeenth century. Least certain is that he drew from manuscript sources. Nevertheless, for all our inability to trace specific loci for specific ideas, we can safely assume that, given its prevalence at the time, he must have been conversant in a general way with Perspectivist theory.

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[29] "Ie n'ay pas peur qu'aucune personne de iugement se persuade que i'aye emprunté ma Dioptrique de Roger Bacon" (*AT,* 2: 447 [ll. 15–17]).

III. DESCARTES'S ANALYSIS OF REFRACTION

Judged by its internal logical structure alone, Descartes's "proof" of the sine-law in the *Dioptrique* is unquestionably a failure because it is glaringly inconsistent at certain points with the theoretical principles upon which it was supposedly built. Our first task, therefore, is to pinpoint these inconsistencies, starting with an analysis of the theoretical principles, which Descartes established in his general theory of light. What was light, and what were its essential physical properties for Descartes? Only by answering these two basic questions can we lay the proper groundwork for reconstructing the entire analysis of refraction upon which his "proof" turns. Having completed that reconstruction, we can then turn to a critical evaluation of his argument, from presuppositions to conclusions, in order to see precisely where and how its major logical flaws emerge.

Cartesian Light-Theory

Early in the *Dioptrique* we are invited by Descartes to compare sight with the action of a blind man tapping out a path between obstacles with a cane or walking-stick.[1] Just as the blind man feels his way by means of physical shock or pressure transmitted from surrounding objects through the cane to his hand, so we "feel" our way visually by equivalent means of shock or pressure passed through any transparent medium from luminous sources to the eye. Accordingly, Descartes urges us to:

> consider light as nothing else, in bodies that we call luminous, than a certain movement or action, very rapid and very lively, which passes toward our eyes through the medium of the air and other transparent bodies, in the same manner that the movement or resistance of the body that this blind man encounters is transmitted to his hand through the medium of his stick.[2]

"This," he concludes, "will prevent [us] at the outset from finding it strange that this light can extend its rays in an instant, [since] the action with which we move one of the ends of a stick must thus pass in an instant to the other end."[3]

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[1] *AT,* 6: 83–84. The use of this sort of analogy to explain vision harks back to Stoic theories of sight (see Lindberg, *Theories of Vision,* 9–11).

[2] "Ie desire que vous pensiés que la lumiere n'est autre chose, dans les corps qu'on nomme lumineux, qu'vn certain mouuement, ou vne action fort promte et fort viue, qui passe vers nos yeux, par l'entremise de l'air et des autres corps transparens, en mesme façon que le mouuement ou la resistence des corps, que rencontre cet aueugle, passe vers sa main, par l'entremise de son baston" (*AT,* 6: 84 [ll. 14–22]).

Descartes's point is admirably clear: we must understand that light is neither a formal attribute of luminous bodies nor a qualitative state in transparent media. Light is, instead, a sort of outward impulse or push exerted continuously by luminous objects upon the transparent medium and simultaneously passed through it to the eye. With respect to any given luminous object, the medium can thus be conceived of as a continuous enveloping body, or walking-stick. Through it the luminous pressure from every point on the light-source will be constantly transmitted, as long as source and medium are in contact. That, Descartes assures us, is why "there is no need to assume that something material passes from the objects to our eyes to make us see colors and light, nor even that there is anything in those objects which is similar to the ideas or sensations that we have of them."[4]

Using the walking-stick analogy, Descartes has only provided an exemplary model for his conception of light. He has by no means offered a systematic theoretical justification. Throughout the *Dioptrique,* in fact, he pointedly eschews theoretical considerations.[5] However, this is not to deny that he had them in mind when framing that account. On the contrary, there is every reason to suppose not only that he did, but that those very considerations underlie the cosmogonic scheme outlined in the first part of *Le Monde,* which is aptly subtitled *Traité de la lumière* ("Treatise on Light").[6] The universe Descartes describes there consists of three fundamental elements—Earth, Air, and Fire—which are differentiated not by intrinsic Aristotelian qualities (such as heaviness and lightness, moistness and dryness, or hotness and coldness), but according solely to the size, shape, and arrangement of their constituent particles, each of which is perfectly inelastic.[7] Those particles that form Earth are relatively large and irregular in shape. Air is composed of relatively small spherical particles. And Fire consists of particles that are so small and varied in shape that they fill the interstices between the particles of Air, which in turn occupy the larger gaps left by imperfectly joined particles of Earth.[8] Thus, in Descartes's universe there are no void spaces whatever, from which it follows that no body within it can act independent of the action or reaction of surrounding, contiguous bodies.[9]

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[3] "Ce qui vous empeschera d'abord de trouuer estrange, que ceste lumiere puisse estendre ses rayons en vn instant, depuis le soleil iusques a nous: car vous sçaués que l'action, dont on meut l'vn des bouts d'vn baston, doit ainsy passer en vn instant iusques a l'autre" (*AT,* 6: 84 [ll. 22–27]).

[4] "Il n'est pas besoin de supposer qu'il passe quelque chose de materiel depuis les obiects iusques a nos yeux, pour nous faire voir les couleurs et la lumiere, ny mesme qu'il y ait rien en ces obiects, qui soit semblable aux idées ou aux sentimens que nous en auons" (*AT,* 6: 85 [ll. 14–19]).

[5] At the outset of the *Dioptrique,* Descartes specifically (and perhaps disingenuously) denies that he intends in any way to explain the actual nature of light. It is enough for his purposes, he claims, to supply some handy and readily apprehensible comparisons with the clear understanding that, like the astronomers, he is making no claims for the truth of his model (see *AT,* 2: 83 [ll. 11–27]). For two recent discussions of Descartes's use of analogies as vehicles of persuasion, see Bruce S. Eastwood, "Descartes on Refraction: Scientific versus Rhetorical Method," *Isis* 75 (1984): 481–502, and Peter L. Galison, "Descartes's Comparisons: From the Invisible to the Visible," *Isis* 75 (1984): 311–26; see also Galison, "Model and Reality in Descartes' Theory of Light," *Synthesis* 4 (1979): 2–23.

[6] Although it was not actually published until 1664, long after Descartes's death, we have every reason to suppose that *Le Monde* was planned and well underway before Descartes undertook to write the *Dioptrique.* In a letter of late 1630 to Mersenne, Descartes not only adverts twice to "mon Monde" with reference to the *Dioptrique,* but characterizes the *Dioptrique* itself as "vn abregé" (*AT,* 1: 179).

Once set in motion, such a tight press of bodies, denied the possibility of free, rectilinear motion through space, will begin to swirl, forming a vortex. Each component particle within that vortex will move with a certain speed that depends upon its size.[10] For example, the large particles of Earth, being commensurately resistant to motion, will be relatively sluggish. The particles of Air will move more freely and swiftly according to their smaller size, and the yet smaller particles of Fire will move even more freely and swiftly. Eventually, through relative displacement, these Fire particles will percolate downward toward the center of the vortex where they will coalesce into a spherical pocket of luminosity enveloped by a huge sphere of Air in which planets and comets ride.[11] From such central pockets of Fire the stars, including our own sun, are formed. Because of the relative smallness and swiftness of their constituent particles, these luminous spheres swirl more swiftly than the Air surrounding them. Consequently, the particles at their surfaces develop a centrifugal tendency that causes them to push outward against the Air that presses in upon them from all sides. This constant outward pressure, which is dispersed uniformly throughout the enveloping Air, is light, whereas the physical capacity of the Air to accept and transmit such pressure amounts to transparency.[12]

Given this account, we can (as Descartes encourages us to do) readily picture Air as a pervasive, crystalline fluid that fills the space between luminous sources, such as the sun, and crass physical bodies, such as planets and eyes. Indeed, so subtle and pervasive is Descartes's Air that it does not just fill space; in a sense it *is* space—or at least what we perceive as space.[13] For all practical purposes, then, Air forms a perfect continuum through which uniform spheres of luminous pressure are generated by light-sources. That pressure is exerted outward in all directions along innumerable vectors or rays. Meantime, by virtue of its rectilinearity, each of these vectors constitutes the shortest possible line of pressure-distribution from any given point on the luminous surface to any given point in the medium.[14]

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[7] See *AT,* 11: 24–25, for a description of the three elements; 11: 25 (l. 25)–26 (l. 26), for a discussion of qualities; and 11: 17 (ll. 9–21) for the claim of perfect inelasticity (i.e., solidity and incompressibility). For a recent sketch of Descartes's cosmological scheme in *Le Monde,* see John W. Lynes, "Descartes' Theory of Elements from *Le Monde* to the *Principes,*" *Journal of the History of Ideas* 43 (1982): 55–72.

[8] Descartes's Air should not be confused with the air we breathe, which is really composed of crass, earthy particles according to the Cartesian system. By Air he meant to specify what would later be called Ether.

[9] *AT,* 11: 37 (ll. 15–23). Although it is a true plenum, Descartes's universe is nonetheless subdivided into constituent particles. For that reason, his theory of matter is usually styled "corpuscularist" to distinguish it from true atomism, which is contingent on the assumption of void space.

[10] *AT,* 11: 48–53.

[11] *AT,* 11: 52 (l. 22)–53 (l. 6).

[12] *AT,* 11: 53 (ll. 6–12); see also *AT* 6: 88 (ll. 15–24).

Although adequate at a macroscopic level, this understanding of Descartes's physics of light is somewhat wide of the mark at a microscopic level. For, as we have just observed, Descartes's Air has a definite internal structure, being composed of tightly packed, totally rigid spherical particles that abut only at points of tangency. This means that the enveloping body of Air is not really continuous at all. Structurally, in fact, it is a only a *virtual* continuum formed by actually discrete, but contiguous parts. Within such a matrix, the impulse that produces light will be transferred instantly through points of contact, sphere by sphere, in a true spatial succession. Thus, the ray seems to represent a real, discrete line of luminous pressure transmitted along any rectilinear succession of contiguous Air particles.

When we examine the model more closely yet, we find that even this conception of the ray is inadequate. It turns out that light can be transmitted in straight lines through *any* array of Air particles, not just a rectilinear succession. For instance, using Figure 2, let us assume that the original pressure from point X on the luminous surface is exerted against particle A of Air along vector XA. That impulse will naturally be distributed evenly between particles B and C along vectors AB and AC. Continuing through the rectilinear succession of particles B, B′, B″, B‴ and C, C′, etc., the impulse will yield rays AO and AP. Yet, once given impulse AB, particle B will also distribute it to particle D along vector BD and to particle K along vector BK, and so forth. By such means, we can generate a variety of pressure-distributions other than AO and AP, such as ABDEFGHI or ABKLMN. While not rectilinear themselves, they will still yield average or mean distributions along rectilinear vectors AI and AN. It is these imaginary lines of mean distribution rather than the tortuous lines of actual impulse-transfer which constitute rays.[15]

It follows therefore that the mathematical ray, being perfectly straight, represents the shortest, most efficient possible line of light-transmission, for, "while Nature has several ways to arrive at a given effect, she always

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[13] *AT,* 11: 29 (ll. 11–17), and 11: 30 (ll. 3–7).

[14] *AT,* 11: 98 (l. 26)–99 (l. 3); 11: 100 (l. 25)–101 (l. 11); see also *AT,* 6: 86–88.

[15] *AT,* 11: 99–100. The basic illustration Descartes uses in this case is that of a cane bent in a serpentine fashion; the pressure passed from one end to the other must travel through all the particles and must therefore follow the tortuous path dictated by the cane's shape, but it will be felt in a direct line with the point of application. See also *AT,* 6: 86–87 and 2: 77 (ll. 1–6).

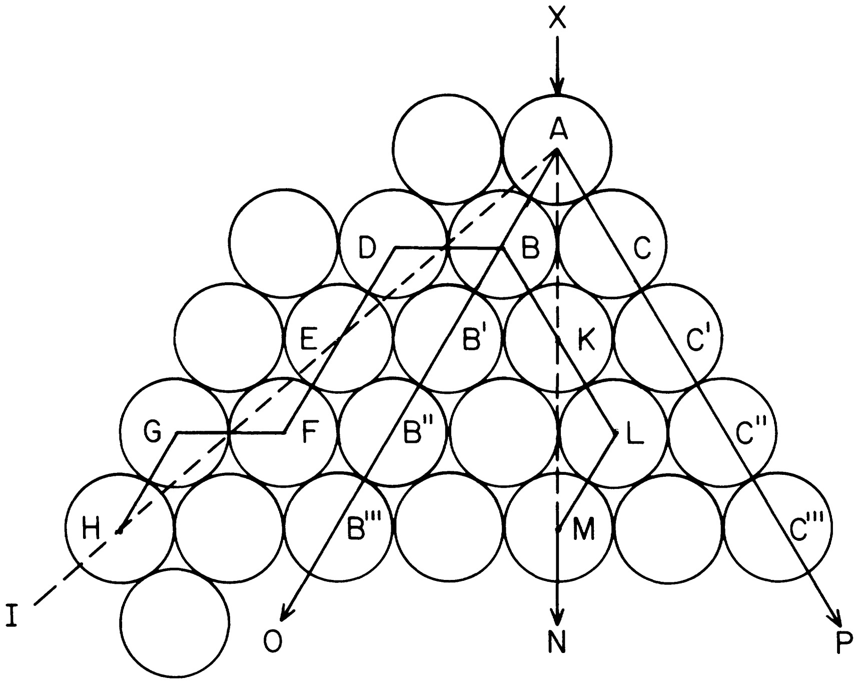

FIGURE 2

[and] infallibly follows the shortest."[16] In addition, the more particles we take into account, the more lines of mean distribution will emerge. Therefore, since the number of Air particles within any perceptible quantum of space surrounding a luminous point is virtually unlimited, so is the number of possible lines of mean pressure distributed from such a point through that quantum. Even though it occurs within a matrix of actually discrete particles, then, Cartesian light-radiation can be effectively treated as both continuous and spherical.

Perhaps it is already evident that, under the conditions set forth by Descartes in *Le Monde,* light can be considered not just as an exertion of physical pressure upon the aereal medium, but also as motion under total constraint, immediately aborted by the unyielding press of Air that prevents the very first particle from flying off when it is given a centrifugal nudge at the point of contact with the luminous surface. Bear in mind, however, that the overall constraint posed by the surrounding medium can be resolved into as many individual lines of constraint as there are corresponding rays or lines of pressure-distribution, so each of these lines of constraint

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[16] *AT,* 11: 89: "Lors que la Nature a plusieurs voyes pour parvenir à vn mesme effect, elle suit toujours infailliblement la plus courte" (ll. 16–18).

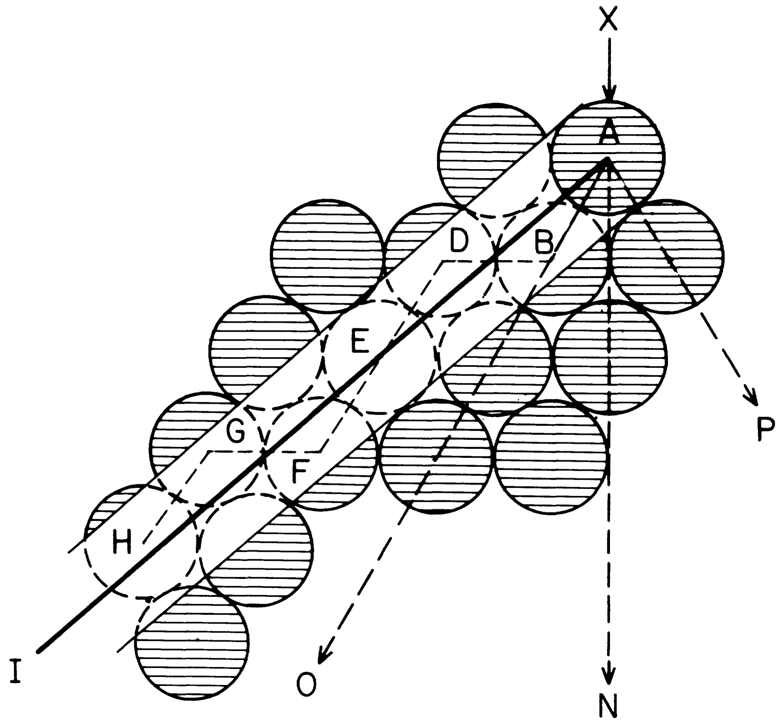

FIGURE 3

represents a potential trajectory for the Air particle after it has been nudged outward. Precisely what this means is illustrated in Figure 3. Normally, luminous impulse XA upon particle A will be instantly dispersed in all directions through the surrounding medium, both the instantaneity and the omnidirectionality of that dispersal being due to the absolute rigidity of the pack enveloping A. Yet, if a particle-wide channel (centered, say, on ray AI) were abruptly evacuated to free A of all interference in that direction, A would obviously shoot off along AI with whatever impetus was originally imparted to it. Ray AI therefore represents the trajectory that particle A would follow if it were at liberty to move in this direction, and the same holds for AO, AN, and AP. Each represents a possible trajectory for that particle.[17] On that understanding, it is not unreasonable to conclude with Descartes that:

> the action, or *inclination to move,* which is transmitted from one spot to another in the Air through the mediation of numerous interjacent bodies lying uninterrupted within the entire intervening space, follows precisely that course along which the

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[17] This illustration is not supplied by Descartes as I present it; it is instead an extrapolation from his discussion in *AT,* 11: 91–94.

same action would cause the first of these bodies to move, if the others were not in the way.[18]

In other words, even though it is only potential movement, light will follow the rules of actual movement.

Appealing finally to intuition, Descartes has achieved two crucial ends with this example. First, he has led us to concur that the ray, though not actually a trajectory, is nonetheless just like one. Second, and far more important, he has effectively equated the action of any given luminous impulse with the motion it would produce along such a trajectory. In short, he has established a virtual dynamic equivalence between the radiation of light and projectile motion. As we shall soon see, this fundamental equivalence gave him license to reduce the physical activity of light to terms of actual material impact, rebound, and deflection.

The Analysis of Refraction

Any discussion of the Cartesian analysis of refraction must begin with Ptolemy. For, although he himself failed to resolve the fundamental problem of refraction, Ptolemy nonetheless furnished the analytic keys which, once turned by the Perspectivists, opened the way for its eventual resolution by Descartes.[19] The central issue is precisely how the line of sight—whether established by visual rays or light rays—is bent or deflected when it meets the interface between two different transparent media, such as air and water.[20] Ptolemy's response was contingent on two basic assumptions. The first of these was that the deflection is caused by a sort of physical resistance, as if the ray were suddenly impeded and then turned at the interface the way a physical body would be when passing from air to water. Thus impeded, the ray would naturally tend toward the normal (i.e., the line perpendicular to the interface of the two media at the point of impact).[21] The second assumption (which follows almost automatically from the first) was that such impedance is due to a difference in density between the two media and, consequently, that the greater this difference, the greater the deflection. Reflection thus becomes a special case of refraction in which the density-differential is so great that the ray is forced to rebound because it is no longer able to break through the interface.[22]

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[18] "L'action ou *l'inclination à se mouvoir,* qui est transmise d'vn lieu en vn autre, par le moyen de plusieurs corps qui s'entretouchent, et qui se trouvent sans interruption en tout l'espace qui est entre deux, suit exactement la mesme voye, par où cette mesme action pourroit fair mouvoir le premier de ces corps, si les autres n'estoient point en son chemin" (*Le Monde,* chap. 14 [*AT,* 11: 102 (l. 22)–103 (l. 1)]—my emphasis in both text and translation. See also *AT,* 6: 89 [ll. 1–4]).

[19] Ptolemy's analysis of refraction occurs in the fifth book of his *Optics,* which exists in a critical Latin edition by Albert Lejeune, *L'Optique de Claude Ptolémée* (Louvain: Université de Louvain, 1956)—henceforth, all references to Ptolemy's *Optics* will be from this edition. An English translation of certain parts of Ptolemy's refraction-analysis can be found in Morris Cohen and I. E. Drabkin, *A Source Book in Greek Science* (Cambridge: Harvard University Press, 1965), 271–81. For a discussion of Ptolemy's attempt to resolve the problem of refraction, see my "Ptolemy's Search for a Law of Refraction: A Case-Study in the Classical Methodology of 'Saving the Appearances' and its Limitations," *Archive for History of Exact Sciences* 26 (1982): 221–40."

[20] Ptolemy's optical theory was based on the assumption that the eye emits a visual flux from its center along straight lines. Thus, in his analysis, what is refracted is not light but this visual flux. Nevertheless, the visual-ray theory and the light-ray theory are mathematically equivalent.

[21] See *Optics,* book 5, sections 31 and 33, pp. 243–44.

Together, these two assumptions point toward a very simple explanation or model of refraction. Each transparent medium is to be thought of as a fluid whose internal structure is more or less tightly packed, but always loose enough to let light through. For its part, light (or the visual flux) is to be thought of as a corpuscle or succession of corpuscles penetrating radially through such fluids. The ease of penetration will be a function of the tightness of the medium's internal structure, so that when light passes from a looser to a more tightly packed medium its penetration will be commensurately weakened or retarded. That weakening will in turn cause the light to be deflected proportionately toward the normal, and the amount of deflection will be consistent with the density-differential. Accordingly, when passing from one transparent medium to another, the ray should always be bent to the same extent. To put it more precisely, given a constant density-differential, there should be a constant mathematical relationship between the angle of incidence and the angle of refraction.[23] The problem for every optical theorist after Ptolemy was to determine exactly what that mathematical relationship is.

The simplest way to understand how Descartes put this model to use in his study of refraction is by recourse to his analysis of reflection. The elements of that analysis consist of a perfectly inelastic projectile (a tennis ball, oddly enough, being Descartes's example) and a totally impermeable surface.[24] When such a projectile strikes such a surface (for instance, at point B along AB, as it is pictured in Figure 4), the result is a complete rebound along BC.[25] The only change that occurs is along the vertical, the downward tendency of the projectile having been converted to a perfectly contrary upward tendency. Nothing else is modified or lost, so the overall momentum or speed of the projectile remains the same throughout. The actual resulting path BC is in fact the precise mirror-image of BH, the path the projectile *would* have taken had it not been impeded at the surface.

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[22] See *Optics*, 5. 1–3, 223–25; see also my "Ptolemy's Search," 230–32.

[23] That this was Ptolemy's understanding is indicated by his claim that the angles of incidence and refraction have "a certain similarity" ("nulla fit in eis flexio ad equales angulos, *sed habent similitudinem quandam et quantitatem que sequitur habitudinem perpendicularium*" [*Optics*, 5.2, 224 (ll. 6–8)—my emphasis]).

[24] The requirement that both the ball and the surface be perfectly solid or inelastic is a function of the need to make the analogy fit the truth as closely as possible. The "truth" for Descartes was that the basic elements of light-transmission (Fire and Air) were perfectly solid and inelastic, so the elements of the analogy had to be too (see Robert S. Westfall, *Force in Newton's Physics* [New York: Neale Watson, 1971], 67–72 and 82–84). This assumption of perfect inelasticity (traduced by the very example that he used—i.e., a tennis ball) was a point of sharp controversy between Descartes and Hobbes (see Sabra, *Theories of Light*, 79–82).

[25] This figure is drawn from Descartes's actual diagram in *AT*, 6: 95.

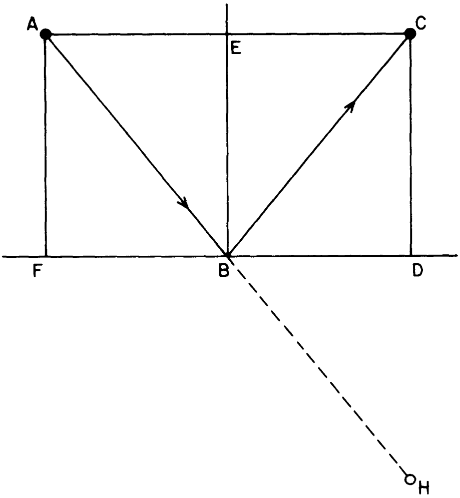

FIGURE 4

What has happened dynamically is that the surface has posed a perfect vertical counter to the downward push of the projectile and thereby converted it to an exactly equivalent upward push.[26] Since no other force or resistance has been exerted, the dynamic crux for reflection must lie in that upward resistance.

So much for what happens when the projectile strikes a perfectly resistive surface. What if, instead of being total, the resistance is lessened enough to let the projectile through? Take the case of a tennis ball striking a flimsy gauze net in lieu of an impermeable surface. Obviously, some of the opposition to penetration would be overcome, and, as we can see in Figure 5, the ball would pass through to a certain extent, depending on the amount of resistance exerted by the net.[27] In other words, the ball will have been only *partially* reflected at point B, so that its downward push will have suffered diminution rather than the total conversion it would have undergone in true reflection.[28] The result will be a swerve away from the normal

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[26] *AT,* 6: 95 (ll. 14–22). Notice that, because both the surface and the projectile are perfectly inelastic, the change occurs instantaneously, without any sort of gradual deceleration and subsequent acceleration caused by compression before rebound. See Jean Wahl, *Le rôle de l'idée de l'instant dans la philosophie de Descartes* (Paris: Vrin, 1953).

[27] See *AT,* 6: 97, for the tennis-ball-and-net analogy and its accompanying figure. The first example Descartes uses involves projection through a thin tissue or net, but later he extends the analogy to projection into water.

[28] As we have just seen, the notion that reflection is just an extreme case of refraction underlies Ptolemy's analysis as well. The rationale behind this notion is simple enough. Light, just like a body, will break through the surface of water when it is projected within a certain angular range. But when that range is exceeded and the angle becomes too oblique, it will be reflected, "ce qu'on a quelquefois experimenté auec regret, lorsque, faisant tirer pour plaisir de pieces d'Artillerie vers le fons d'vne riuiere, on a blessé ceux qui estoyent de l'autre costé sur le riuage" (*AT,* 6: 99 [ll. 25–28]).

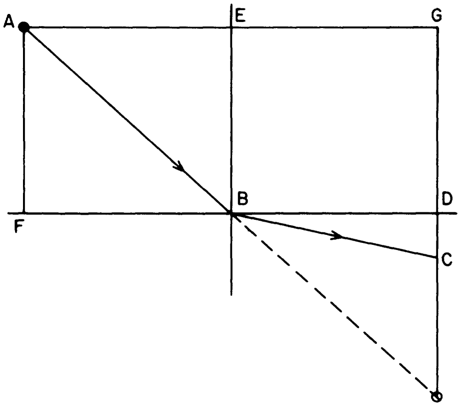

FIGURE 5

EB along BC, and the amount of swerve will be contingent on the amount of resistance posed by the surface at point B. The greater that resistance, the greater the swerve will be; and the greater the swerve, the greater the overall loss in the ball's momentum or speed. Presumably, the same thing that happened in reflection has happened here as well, since this is merely an instance of incomplete or aborted reflection—that is, the only force exerted is the upward force of resistance applied at point B. Accordingly, the only real dynamic change to have occurred is the one caused by that force: namely, a loss in the downward momentum of the projectile.[29] Everything else remains the same, so that, although there seems to have been an overall loss in momentum or speed, it is due solely to the loss of force or momentum along the vertical.

The significance of this dynamic analysis becomes graphically clear when we follow Descartes's vectorial analysis of the situation. According to him, the basic motions along AB and BC can be resolved into perpendicular and horizontal components or "determinations."[30] That these components remain perfectly equal—horizontal to horizontal and vertical to vertical—after impact and *reflection* is clear from Figure 4. After all, AE is equal to EC, and AF is equal to CD. In the case of partial reflection, or *refraction,* though, the motion along the vertical diminishes, while its horizontal counterpart remains constant. Thus, in Figure 5, AE is equal to EG, which is equal to BD, and refracted ray BC is shorter than incident ray AB. Likewise, CD is shorter than AF. So the horizontal momentum is conserved throughout, no matter what happens along the vertical.[31]

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[29] *AT,* 6: 98 (ll. 3–5).

[30] Descartes's division of "force" into "determinations" implies a distinction between the projectile's (or light's) absolute force or speed (which is represented by the actual trajectory) and its force or speed relative to horizontal and vertical. Thus, "determination" seems to be a combination of force or speed and direction; and, according to the breakdown by components, it is nothing more than a projection of the absolute speed along the horizontal and vertical (see Ole Knudsen and Kurt Møller Pedersen, "The Link between 'Determination' and Conservation of Motion in Descartes' Dynamics," *Centaurus* 13 [1968]: 183–86). Fermat had enormous difficulties with the notion of "determination," apparently because he thought it had to do with direction only (see Sabra, *Theories of Light,* 84 and 117–21).

An ostensible justification for this particular rule of conservation can be found in Descartes's general "law" of conservation stemming from the first rule of motion in the seventh chapter of *Le Monde*. "Each piece of matter, taken by itself," Descartes asserts there, "continues forever in the same state as long as contact with other [pieces of matter] does not force it to change." Even after contact, he adds, the overall motion is preserved.[32] Evidently, then, despite interaction with other pieces of matter at surface FD, light conserves its overall motion by maintaining its original inertial state along the horizontal. Moreover, as in reflection, so in refraction, the dynamic change occurs instantaneously, whether the ball strikes a thin net or the surface of a fluid. Whatever internal resistance the fluid might have below the surface plays no part at all in the refractive process beyond determining how much vertical opposition to traversal will be posed at the instant of impact.[33] In short, *the dynamic effect of a fluid's density is concentrated entirely at its surface.*

Since there is no loss whatever along the horizontal, the time it takes the projectile to traverse AB in both Figures 4 and 5 must be precisely the same as that required to traverse BC. But AB is longer than BC in the case of refraction (Fig. 5), so the projectile must pass more slowly along BC. Here we have the key to Descartes's solution of the refraction problem, because, according to him, there is an inverse proportionality between the speeds (as measured by ray-lengths AB and BC) and the densities of the two media separated by interface FD.[34] In other words, the greater the density of the medium, the slower the body's progress. Figure 6 serves to illustrate this point. If the medium above interface FD were half as dense as the one below it, the time it would take a projectile to traverse BC, which is half as long as AB, would be equal to the time it took that same projectile to traverse AB. It would therefore take the projectile the same amount of time to traverse horizontal components FB and BD.[35] By extension, the time it would take to traverse BI would be twice the time it would take to

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[31] This follows naturally from the assumption that, just as in reflection, so in refraction, the only force at work upon the projectile is the resistance to penetration posed by the surface along the vertical.

[32] "Chaque partie de la matiere, en particulier, continuë toujours d'estre en vn mesme estat, pendant que la rencontre des autres ne la contraint point de la changer" (*AT*, 11: 38 [ll. 9–12]). Descartes adduces two additional laws from this: first, that in the case of collision the overall motion is conserved (*AT*, 11: 41 [ll. 1–5]); and second, that all bodies, no matter the shape of their actual trajectory (e.g., in circles), have a constant and natural tendency to move rectilinearly (*AT*, 11: 43 [l. 26]–44 [l. 7]).

[33] *AT*, 6: 98 (l. 29)–99 (l. 9).

[34] *AT*, 6: 98 (ll. 15–29).

[35] *AT*, 6: 97 (l. 29)–98 (l. 14). AF in Figure 6 represents the *cathetus*—i.e., the perpendicular dropped from point A to FG—so it is parallel to normal EB. Thus, FB must be equal to AE, the direct measure of the horizontal component. CD and IG likewise are *catheti*, so BD and BG represent the horizontal components of motion along BC and BI respectively.

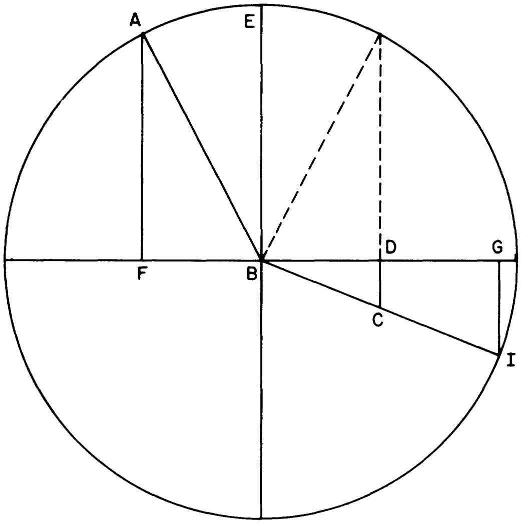

FIGURE 6

traverse AB, since AB is equal to BI (both being radii of the same circle). The time of traversal along horizontal component BG must therefore be double the time of traversal along FB, and BG must be twice as long as FB. It follows that, whenever a physical projectile moves from a rarer to a denser medium, its speed will be commensurately less. It also follows that its new trajectory will be away from the normal in proportion to the dictates of horizontal conservation. Conversely, whenever a projectile moves from a denser to a rarer medium, its speed will increase commensurately, and its new trajectory will incline toward the normal in proportion to the dictates of horizontal conservation.[36]

When we attempt to apply this model of physical projection and impact to refraction, we encounter a problem. If we assume with Descartes that light acts just like a physical projectile, then when it passes from a rarer to a denser medium and undergoes impedance at the interface of the two, its new path ought to incline away from the normal. But in fact the very opposite occurs. The new path inclines *toward* the normal. Faced with this obvious inconsistency, we must suppose with Descartes that the dynamics of light are exactly contrary to the dynamics of physical projection and, consequently, that light actually *gains* momentum (or, as he puts it, finds "easier passage") when it enters a denser medium.[37] This easier passage,

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[36] *AT,* 6: 100 (l. 27)–101 (l. 5). This principle of reciprocity was taken for granted by Ptolemy and every optical theorist dealing with refraction after him (see Ptolemy, *Optics,* 5.31, 243).

[37] In this case, the tennis-ball-and-net analogy requires that a player be stationed at the net so that, as soon as the ball strikes the net, he can slam the ball with his racket toward the normal in order to give it added force after impact (*AT,* 6: 99 [l. 29]–100 [l. 9]). It should be noted that Descartes is sedulous throughout his analysis in the *Dioptrique* not to use the term

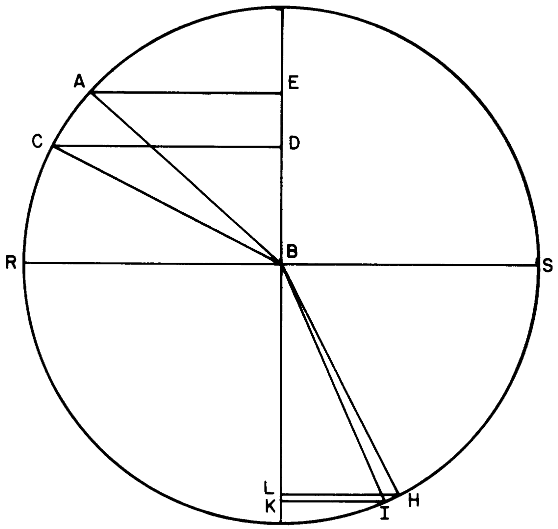

FIGURE 7

he explains, is due to the relative tightness or looseness of the medium's internal structure "such that, the harder and more firmly packed the tiny [constituent] parts of a transparent body are, the more easily they allow the light to pass."[38] Thus, granted that light does pass more easily through denser media, the erstwhile *inverse* proportionality between momentum or speed and density must be transformed into a *direct* proportionality. Once that direct proportionality is granted, the grounds for Descartes's mathematical derivation of the sine-law are completely laid.

The proof that Descartes constructs upon these grounds is essentially as follows: let the light traveling along rays AB and CB in Figure 7 strike the denser medium's surface RS at point B and, having gained a certain amount of speed, proceed along rays BI and BH refracted toward the normal.[39] Assume that the speed in the denser medium is twice that in the rarer medium—i.e., in a ratio of 2 to 1. Therefore, the light will traverse the same distances BI and BH in half the time required to traverse AB or CB, all four lines being radii of the same circle. But if the speed or "determi-

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speed ("vitesse") when talking about light. Rather, he depends on the terms "force" and "passage," which connote, but do not actually denote, temporal process. As early as 1619 Descartes had already concluded that light penetrated denser media more easily than it did rarer ones (see *Cogitationes privatae* [*AT,* 10: 242–43]).

[38] "En sorte que, d'autant que les petites parties d'vn cors transparant sont plus dures et plus fermes, d'autant laissent elles passer la lumiere plus aysement" (*AT,* 6: 103 [ll. 25–27]). In order to help us grasp this idea of an easier passage through a more rigid medium, Descartes urges us to compare the ease with which a hard ball rolls along a firm, unyielding surface (e.g., a bare table-top) to the difficulty with which it rolls along a soft, yielding surface (e.g., a carpet). Obviously, the ball will roll faster along the first surface than it will along the second (see *AT,* 6: 103 [ll. 16–24]).

[39] See *AT,* 6: 102, for Descartes's figure.

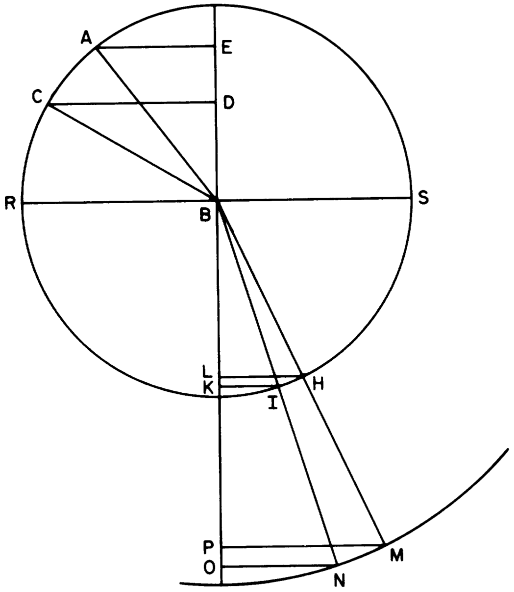

FIGURE 8

nation" along the horizontal remains the same throughout, the horizontal determination AE will necessarily be twice the horizontal determination KI, because the time of traversal along the former was twice that along the latter. Likewise, CD will be twice LH, and these horizontal components are as the sines of their angles (i.e., AE is directly proportional to angle ABE, and so forth). Hence, the ratios AE:KI and CD:LH will be equal.[40] This proportionality will hold in every case, so that sine *i*:sine *r* will always be constant, no matter what the original angles or what the difference in density between the two media, provided only that the speeds along the incident and refracted rays are directly proportional to those densities.

Another, perhaps more immediately evident, way or arriving at the same conclusion is illustrated in Figure 8. Assume that line RS separates two media and that the upper medium is twice as rare as the lower one. Assume, further, that light travels through the upper medium along ray AB and, after striking the interface at B, follows a new radial path BN inclined toward the normal EO. According to the assumption of direct proportionality between speed and density, the light will travel twice as far through the denser medium as it did through the rarer in the same amount of time. However, from the previous vectorial account, we know that the light's horizontal component will remain unchanged. Therefore,

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[40] The actual sine of any angle is equal to the opposite divided by the hypotenuse, but since each hypotenuse in this case (i.e., AB, AC, BH, and BI) is a radius of the same circle, it can be dropped out of the account, leaving merely the opposite sides to be compared.

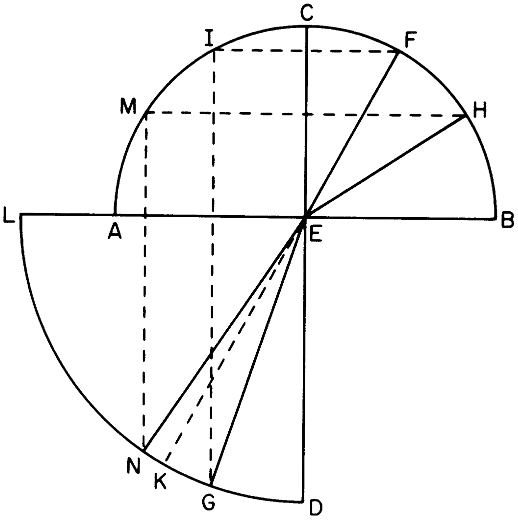

FIGURE 9

AE and ON will be equal. The same holds for rays CB and BM, whose horizontal components CD and PM will be equal. Now, since refracted rays BN and BM cut circle ARS at points I and H respectively, the respective horizontal components for BI and BH will be KI and HL. But ON:KI = PM: HL, and ON = AE, while PM = CD. Consequently, AE:KI = CD:HL, and since all four lines are as the sines of their respective angles, it is obvious that sine *i*:sine *r* will be constant throughout.[41]

That this may well have been the actual path of reasoning Descartes followed to his final proof in the *Dioptrique* is borne out by the strikingly similar approach that his close friend and collaborator, Claude Mydorge, took to the problem some eleven years earlier:

> *Given the* [*angles of*] *incidence and refraction of any ray* [*inclined*] *upon the surface of whatever transparent body you wish, to find the* [*angle of*] *refraction for all other rays inclined upon the same surface.*
>
> Let there be a plane, upon which A, E, and B [in Figure 9] lie, such that it forms the common section of some [transparent] body and whatever plane you wish.

---

[41] According to the construction in Figure 8, the actual proof of the sine-relationship follows quite easily from Euclidean geometry:

Given: BH = BI and BM = BN,
AE = NO and CD = MP,
To Prove: AE:KI = CD:HL.
1) BH:BM = BI:BN (*Elements,* book 5, prop. 14);
2) BH:HL = BM:MP and BI:KI = BN:NO (*Elements,* 6.2), so
3) BH:BM = HL:MP and BI:BN = KI:NO (*Elements,* 5.16); but
4) BH:BM = BI:BN (by step 1), so
5) HL:MP = KI:NO; but
6) MP = CD and NO = AE (by construction), so
7) HL:CD = KI:AE. QED

> Accordingly, let straight line AEB be the common section of the surface [of refraction], and let inclined ray FE be refracted toward G. When perpendicular CED is drawn, CEF will be the angle of *incidence* for ray FE, and KEG will be the *angle of deviation* for that same ray [which is to say that GED will be the angle of refraction].
>
> If you choose some other ray, such as HE, [which is] incident according to angle CEH, then you will have its [angle] of refraction in the following way:
>
> Let semicircle ACB, of radius EB, be drawn about center E so that its circumference cuts ray FE at point F and ray HE at point H. Having drawn line FI parallel to line AB, let IG be drawn parallel to CE so as to meet refracted ray EG at point G. Then let the circumference of circle LGD drawn with radius EG about center E meet diameters CE and BE prolonged to points L and D. Then, after HM is drawn parallel to BA, let MN be drawn parallel to CD so as to meet circumference [LGD] at point N. Line EN will therefore be the refraction of ray HE broken at point E and will form the *refracted angle* DEN.[42]

Two things are especially noteworthy about this mathematical construction. First, refracted rays EG and EN are explicitly made longer than incident rays HE and FE, which implies that the light has traveled farther—and thus faster—through the denser medium. Second, since FI and HM are bisected by the normal CD, and since IG and MN are parallel to normal CD, the perpendiculars drawn from G and N to CD will be equal respectively to the perpendiculars drawn from I and M to CD. Therefore, if semicircle ACB were completed, it would cut EG and EN in such a way as to yield the sine-law.

In sum, Descartes's derivation of the sine-law rests on six fundamental suppositions, the first being that refraction is a dynamic phenomenon whose specific cause is a change in the momentum or speed of light when it passes through a density-interface. The second supposition is that this change is completed in an instant at the surface of refraction. The third supposition is that light gains momentum or speed when it enters a denser medium, from which it follows, fourth, that the light's momentum or speed along the actual trajectory is directly proportional to the density of the medium through which it passes. The fifth supposition is that the fundamental direction of dynamic change in refraction is along the vertical; and the final one is that the momentum or speed of light along the horizontal is perfectly conserved throughout, despite the dynamic change that occurs along the vertical.

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[42] Letter to Mersenne, 1626, in *Correspondance du P. M. Mersenne*, ed. C. de Waard, et al., 19 vols. [Paris: 1945–83], 1: 404–405:

> *L'Inclination et la refraction de quelconque rayon sur la surface de quelque corps diaphane que l'on voudra estant donnée, donner la refraction de tous les autres rayons inclinez sur la mesme surface.*
>
> Que le plan, où se trouvent A, E, B soit la commune section de quelconque corps et de tel plan que l'on voudra, et partant que la droite AEB soit la commune section de la surface; et que la refraction du rayon incliné FE se face en G. Ayant tiré la perpendiculaire CED, CEF sera l'*angle de l'inclination* du rayon FE et KEG sera l'*angle de la refraction* de ce mesme rayon.
>
> Si l'on choisit tel autre rayon qu'on voudra comme HE, incliné suivant l'angle CEH, l'on aura sa refraction en cette manière:
>
> Soit descrit du centre E et de quelconque intervale EB le demi-cercle ACB, dont la circonference coupe le rayon FE au point F et le rayon HE au point H. Et après avoir tiré la ligne FI parallele à la ligne AB, soit tiré IG parallele à CE, laquelle rencontre le rayon rompu EG au point G. Et que la circonference du cercle LGD descrite de centre E et de l'intervale EG, rencontre les diamètres CE, BE prolongez aux points L et D. Et aprez avoir tiré HM parallele à BA, soit descrite MN parallele à CD qui rencontre les circonferences au point N, la ligne EN sera la refraction du rayon HE qui s'est rompu au point E et qui fait l'*angle rompu* DEN.

In referring to KEG as "l'angle de refraction," Mydorge was following the medieval usage of the term *angulus refractionis,* which translates properly to "angle of deviation."

A Critical Evaluation

Whatever its ostensible simplicity and ingenuity, Descartes's mathematical derivation—or "proof"—of the sine-law suffers from a variety of procedural flaws. So flawed is it, in fact, that we might be tempted to pass it off as mere sleight-of-hand, dismissing it, like Richard Westfall, as "preposterous."[43] First, there is a wealth of glaring inconsistencies between the dynamic model upon which Descartes based his analysis and the actual theory of light he proposed at the outset of the *Dioptrique.* How, for instance, could he possibly treat light as matter in motion after having so punctiliously shown that it entails neither material flux nor spatio-temporal passage? Indeed, so critical was the assumption of instantaneous light-propagation in Descartes's view that, were the contrary to be shown, he would have to concede "that my entire [natural] philosophy will be wiped out to its very foundation."[44]

More to the point, how could Descartes relate the speeds through various media of something that is in reality transmitted instantaneously?[45] The only answer he could provide is that the "motion" he attributed to light is virtual, not real. In other words, he was basing his dynamic interpretation of refraction upon the movement that light (in the form of Air particles)

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[43] "As a demonstration, [Descartes's] argument was preposterous. Its foundation consisted of assumptions which were arbitrary and contradictory." Richard S. Westfall, *The Construction of Modern Science* (New York: Wiley, 1971), 53.

[44] "Si quae talis mora [when light passes through space] sensu perciperetur, totam meam Philosophiam funditus eversam fore inquiebam" (letter to Beekman, 22 August 1634, *AT,* 1: 308 [ll. 18–20]).

[45] The source of this apparent absurdity lies in Descartes's having equated movement proper with *tendency* to movement, an equation that was rejected by, among others, Fermat and Morin. Some scholars, recognizing the ambiguity of the term "instant," have pointed out that it could as easily mean "an imperceptible period of time" as "no time at all." That Descartes himself was aware of this ambiguity is clear from the following quotation, in which he dispels all doubt about his meaning: "Et pour la difficulté que vous trouuez," he wrote to Mersenne in May 1638, "en ce qu'elle [light] se communique en vn instant, il y a de l'équiuoque au mot d'instant; car il semble que vous le considerez comme s'il nioit toute sorte de priorité, en sorte que la lumiere du Soleil pust icy estre produite, sans passer premierement par tout l'espace qui est entre luy et nous; au lieu que *le mot d'instant n'exclud que la priorité du temps, et n'empesche pas que chacune des parties inferieures du rayon ne soit dependante de toutes . . . ses parties precedentes*" (*AT,* 2: 143 [ll. 9–20]—my emphasis). For Descartes, then, the term "instant" specifically *excludes* temporal succession while allowing for spatial succession in light-propagation.

would have if it were able to move freely through space. Thus, if light can be reasonably likened to a projectile, it ought to *act* like a projectile. But even if we grant Descartes this license, how can we be certain that the laws governing real, physical motion are the same as those governing the virtual motion of light? We cannot. We can only assume such an identity, which leaves the analogical foundations of Descartes's account shaky from the beginning.[46] They become even shakier when the analogy is extended to fluids, because we know that projectiles shot into water face opposition not only from the surface, but also from the internal resistance below the surface.[47]

The weakness of the physical analogy which is so central to Descartes's account of refraction is bad enough. Worse, his pivotal claim that light speeds up (or finds an easier passage) in denser media is wholly inconsistent with the physics of impact as he described it. Descartes was quite willing to concede that, when striking a denser medium, a physical projectile necessarily *loses* momentum or speed; yet in refraction the precise opposite happens.[48] Light therefore violates the very rules of motion and impact that are supposed to govern it. Moreover, Descartes's attempt to defend the claim for a gain in momentum on the basis of closer packing makes no sense whatever in light of his plenistic theory of matter. The Air that transmits light-impulses is already as tightly packed as possible insofar as its tiny constituent parts are always and necessarily perfectly inelastic and contiguous throughout.[49]

Yet another difficulty is that the principle of conservation of motion along the horizontal, upon which his demonstration finally turns, seems wholly arbitrary, in spite of its purported justification in the rule of "inertia." Common sense would seem to militate against its use.[50] Given the basic model of projection into a fluid, it is self-evident that the diversion of the projectile toward the normal must be due at least in part to a horizontal component of resistance, which is hardly consistent with conservation in that same direction. Not only does that raise questions, but the physical illustration Descartes offers in support of horizontal conservation—in his account of what happens to a physical projectile when it enters a dense medium and flies off away from the normal—is absolutely unreasonable. Intuition alone tells us that in such a case the projectile's loss of speed or momentum will cause it to assume a path inclining *toward* the normal. No one who has thrown a rock into a pool of water could seriously think otherwise. And this casts doubt in turn on Descartes's claim that, since light inclines toward the normal when passing from a rarer to a denser medium, it must be gaining, rather than losing, speed or momentum. No wonder that Fermat and others have charged Descartes with, at best, incoherence and, at worst, cheating. There is certainly no question that, once brought into clear focus, these inconsistencies seem to render his entire analysis of refraction senseless and *ad hoc* from beginning to end.

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[46] "Ie doute premièrement, et avec raison," says Fermat in his first outline of objections, "si l'inclination au mouuement doit suiure les loix du mouuement mesme, puis qu'il y a autant de difference de l'vn a l'autre, que de la puissance à l'acte" (*AT,* 1: 357 [ll. 22–25]). This objection is echoed almost verbatim by Morin (see *AT,* 1: 543 [l. 24]–44 [l. 2]).

[47] This problem seems to underly *OF,* 2: 109.

[48] Actually, Fermat did not raise this issue during the debate either with Descartes in 1637 or with his disciples in 1658–62. It was only in a retrospective analysis of the controversy that he adverted to the problem explicitly, claiming that the idea of light finding easier passage through a denser medium "semble choquer le sens commun" (Fermat to M. de ***, *OF,* 2: 485).

[49] This issue was particularly vexing to Morin (see *AT,* 1: 553–56).

[50] This was the main sticking-point for Fermat. Even though he was willing, at least for the sake of argument, to accept *some* sort of conservation in the act of refraction, he was unwilling to concede that it necessarily lay along the horizontal. But that unwillingness sprang from what he saw as a deeper problem: that Descartes's composition of motion according to vertical and horizontal components was totally arbitrary: "sa diuision des determinations au mouuement n'est pas celle qu'il faut prendre, puisque nous en auons d'infinies" (*OF,* 2: 405 [ll. 1–3]). See Sabra, *Theories of Light,* 116–35, for an excellent detailed analysis of Fermat's attempts at refuting Descartes's demonstration.

To deny, as we must, that Descartes's refraction-analysis is consistent or coherent is not, however, to admit that it is *ad hoc* or senseless. In fact, it is anything but, as will become clear in the next three chapters. The point is that, while they defy explanation on purely logical grounds, the inconsistencies of Descartes's account *can* be explained satisfactorily when traced back to their proper historical source. The specific knots to unravel lie in the following crucial suppositions: 1) that light-radiation is *virtually* equivalent to projectile motion, whereas in *reality* it entails no motion whatever; 2) that the dynamic change which causes refraction takes place only at the surface; 3) that refraction is governed by a principle of conservation along the horizontal; and 4) that the bending of light toward the normal when it enters a denser medium is due to a gain in momentum or speed. In time we will see not only that, but also how, these four suppositions, with all their attendant consequences, derive systematically and logically from Descartes's reliance on the Perspectivist account of light and refraction.

IV. THE FOUNDATIONS OF PERSPECTIVIST OPTICS

Descartes obviously intended his new theory of light to supplant the prevailing Scholastic—i.e., Perspectivist—theory of his day. To a certain extent he succeeded. At first impression, the two theories do seem to be radically different, and, as will shortly become evident, the Cartesian alternative is by far the simpler and more readily comprehended of the two. However, this initial impression is misleading. Although Descartes and the Perspectivists were poles apart in their understanding of the actual cause and nature of light, they nonetheless viewed the physical act of light-radiation in essentially the same way. To demonstrate this point we will examine the Perspectivist theory of light with an eye toward establishing the underlying principles that link it to Descartes's theory. This link is far closer than first impressions might lead us to believe.

Perspectivist Light-Theory

The Perspectivist account of light-radiation hinges on the supposition that light is neither matter nor motion, but rather a qualitative state whose source or efficient cause, luminosity, is an inherent attribute of luminous objects.[1] As such, light finds its formal cause in *lux,* which has the power to replicate itself immediately as *lumen* in all directions. *Lumen,* in short, is the physical form or "species" of *lux.*[2] Like all forms in the Aristotelian universe, however, light requires a properly receptive subject or material cause for its actuation. That need is met by any continuous transparent body, such as air or water. So *lumen* is replicated by *lux* whenever and wherever transparency permits, which is to say that *lumen* is the physical

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[1] For a general account of Perspectivist light-theory, see Lindberg, *Theories of Vision,* 58–146. See also my *Witelonis Perspectivae liber quintus,* 18–44 and 58–66, and "Getting the Big Picture in Perspectivist Optics," *Isis* 72 (1981): 578–82.

[2] Bacon makes this distinction between *lux* and its species, *lumen,* clear in *DMS,* pt. 1, chap. 1, pp. 2 (l. 29)–4 (l. 36): "Et, ut in exemplo pateat hec species, dicimus lumen solis in aere esse specimen lucis solaris que est in corpore suo; . . . Et Avicenna dicit tertio *De anima* quod *lux* est qualitas corporis lucentis, . . . *lumen* vero est illud quod est multiplicatum et generatum ab illa luce"—my emphasis (see also Kepler, *Paralipomena* 1 [*GW,* 2: 19 (ll. 37–42)]). Species, Bacon goes on to say, are only "intentions" of what they represent, not true replicas. Thus, *lumen,* like a painting, is only a physical representation of its generating subject (*DMS* 1.1, 4 [ll. 54–56]). That Descartes was well aware of the *lux/lumen* distinction and its meaning is evident from *AT,* 2: 203–205. Moreover, as in the Perspectivist theory, so in his, there is a similar distinction between light as initial generator (*lux* vs. Fire in rapid rotary motion) and light as generated (*lumen* vs. pressure transmitted through Air).

"act" of *lux* in transparent media.[3] That "act" manifests itself in a qualitative transformation of such media from merely transparent to illumined.

Because *lumen* is just a qualification of *lux* in transparent media, its actual replication entails a sudden and complete change in those media, not a gradual passage through them. Strictly speaking, then, the propagation of light is absolutely holistic according to the Perspectivists; that is, it happens everywhere instantaneously.[4] Consequently, the act itself has no real spatio-temporal determination, and thus no real direction or shape. Yet the central tenet of Perspectivist optics—that light radiates spherically outward from luminous sources—seems to fly in the face of this restriction.[5] In fact, it does not for two reasons. First, although the direction of propagation may be spatio-temporally indeterminate, its outwardness is specified by the causal priority of *lux* to *lumen* and of *lumen* to illumination.[6] Second, as an exercise of power, the act of propagation is spatially determined insofar as light's intensity diminishes continuously with distance from the source. If the propagation occurs within a uniform medium, moreover, the decrease in intensity will be both continuous and uniform. Accordingly, a sphere of uniform light-intensity will be created at every given radial distance from the luminous source.[7]

In mathematical terms, the given sphere of propagation is a locus of points equidistant from, and radially linked to, a central point-source. Each link thus represents a distinct line of radiation whose rectilinearity gives it perfect spatial brevity.[8] Taken at face value, though, this mathematical model conflicts with physical theory in two critical respects. First, it implies that the propagation of light is truly discrete, whereas Perspectivist physics demands that it be absolutely continuous. Second, according to that same physics, actual luminosity requires the conjunction of *lux* (formal cause) and physical bodies (material cause). Since physical bodies, unlike points, have spatial dimension, then so must all luminous sources. In short, light must actually emanate not from discrete points but from quantifiable spots or areas.[9]

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[3] Transparency, according to the Perspectivists, is an intrinsic capacity of certain bodies to accept and transmit light without any change in their essential nature ("Corpora diaphana sunt apta penetrationi luminis . . . sine essentiali sui transmutatione" [*WP* 2.4, 64; see also *DA*, bk. 1, chap. 5, proposition 28, p. 17; *PC*, bk. 1, proposition 51, p. 133; and *Paralipomena* 1, appendix (*GW*, 2: 39 [l. 31]–40 [l. 8])). In Aristotelian terms, the intrinsic capacity of those bodies to accept and transmit visible impressions is a potential, which is actualized by the presence of light in them (cf. *De anima* 2.7.418b4–13). So by extension, the potential of *lux* to produce its species *lumen* is actualized only when such a transparent medium is in contact with the luminous surface. Thus, *lumen* is the physical actualization of *lux* in transparent media insofar as such media allow *lux* to realize its potential for self-replication.

[4] "Lumen non impeditum, per totum sibi proportionatum medium in instanti necessarium est deferri" (*WP* 2.2, enunciation, p. 63; see also *Paralipomena* 1.5 [*GW*, 2: 21], and *PC* 1.3, 62). This notion of a "holistic" transmission of visual effects (i.e., light and color) goes back to Aristotle's account of light and color in *De anima* 2.7. Aristotle attempts to make the idea of instantaneous change understandable through the example of the transformation of water to ice, which, according to him, happens all at once rather than by gradual stages (*De sensu et sensato* 6.447a2–4). In fact, not every Perspectivist agreed that light radiates instantly, although those who did not thought that the time it took was imperceptible (see David C. Lindberg, "Medieval Latin Theories of the Speed of Light," in *Roemer et la vitesse de la lumière*, ed. René Taton [Paris: Vrin, 1982], 45–72).

[5] "Et patet quod multiplicatio est sperica naturaliter, quoniam agens undique et in omnem partem et secundum omnes diametros facit speciem suam" (*DMS* 2.8, 156 [ll. 3–5]). For similar sorts of statements, see *WP* 2, supposition 6 and proposition 20, pp. 61 and 68–69; *DA* 1.5.17, 9; *Paralipomena* 1 (*GW*, 2: 19 [l. 23]–20 [l. 2]); and *PC* 1.6, 83.

[6] That is, the succession of formal causes in light-propagation—*lux*, its immediate effect, *lumen*, and *its* immediate effect, illumination—happens to parallel the spatial succession of material causes—luminous source, transparent medium, and the illuminated opaque body. Therefore, although the "outwardness" of the act of radiation is temporally indeterminate, it is spatially determinate (cf. Descartes's similar understanding in chap. 3, n. 45).

[7] See *WP* 2.7, 64–65; *Paralipomena* 1.9 (*GW*, 2: 22); and *DMS* 2.8, 156 (ll. 2–16).

Actually, this last condition poses no real threat to the original point/ray model. To understand why not, let us turn to Figure 10. Suppose that the small circle with center A represents a luminous surface and that concentric circles EG and HL represent outlying spheres of propagation created by it. Construct angle HAL so that arc BD represents a luminous spot of arbitrary size that illuminates truncated wedge BDLH of the surrounding medium, and let that angle be bisected by line ACFK. Arc BD thus constitutes a determinate section of the entire luminous surface commensurate with portion BDLH of the whole sphere of illumination. As is graphically clear from the figure, the intensity or power of illumination within area BDLH must decrease continuously with distance, for the light emanating from BD disperses constantly as it reaches arcs EG and HL, and the size of those arcs increases constantly with radial distance along CFK. Hence, because the diminution of light intensity is directly related to dispersal, it must also be directly related to the distance along CFK.[10]

Now, suppose that subtending lines HA and LA of angle HAL are squeezed together as tightly as possible toward bisecting line ACFK without meeting it. In that case, the endpoints of arcs BD, EG, and HL will converge upon points C, F, and K, respectively, and area BDLH of illumination will dwindle to an unimaginably narrow filament of light. So narrow can we conceive this filament to be, that for all practical purposes it can be rep-

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[8] Within the classical tradition of natural philosophy, the final cause for all natural actions is Nature's avoidance of wasted effort, which is summed up in the stock dictate that Nature does nothing in vain (see, e.g., Aristotle, *De generatione animalium* 2.4.739b20–21, 2.5.741b5, 2.6.744a36–37, and 5.8.788b21–23). Witelo proposes this "Principle of Natural Economy" as an initial postulate for his treatment of light ("Item quod natura nihil frustra agit, sicut nec deficit in necessariis" [*WP* 2, supposition 8, p. 61]). Since light-radiation is a natural action, it must conform to the Principle of Natural Economy, and the way it does is by taking a rectilinear form, since straight lines constitute the shortest point-to-point connection possible (see *WP* 5.18, 198; see also *Paralipomena* 1.4, [*GW*, 2: 20–21], and *DMS* 5.1, 229). For a discussion of the use of the Principle of Natural Economy in classical geometrical optics, see my "Ptolemy's Search" and "Saving the Appearances of the Appearances: The Foundation of Classical Geometrical Optics," *Archive for History of Exact Sciences* 24 (1981): 73–99.

[9] "Lux enim non procedit nisi a corpore, quoniam non est nisi in corpore: unde patet, quia in minima luce, que sumi potest, est latitudo" (*WP* 2.3, 63 [ll. 56–57]; see also *DMS* 2.1, 95, and *PC* 2.18, 169).

[10] See *WP* 2.24, 70; *DMS* 5.3, 246–49; and *PC* 1.17–19, 92–96 (cf. *Paralipomena* 1.9 [*GW*, 2: 22]).

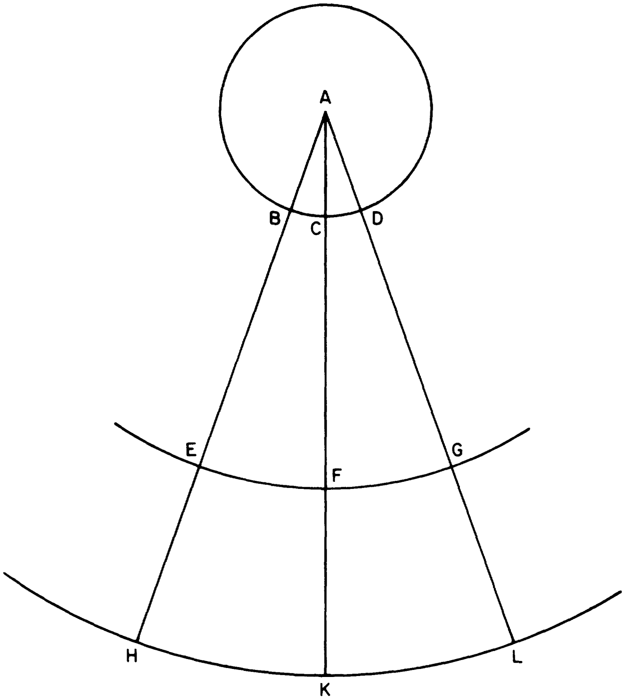

FIGURE 10

resented by line CFK, along which points C, F, and K represent miniscule spots of light. Both the filament and its constituent spots will still have some lateral dimension, though, so what holds for truncated wedge BDLH at the macroscopic level will hold at this level too. Effectively reduced to a point, finally, luminous spot BD will generate its own sphere of propagation composed of innumerable threads of light, each one terminating at a tiny illuminated spot in the surrounding medium.[11]

Far from accepting it as a real physical entity, then, the Perspectivists regarded the mathematical ray as a convenient fiction, an ideal that is only approximated (albeit very closely) in physical reality. It is this point which prompts Witelo to claim that: "in the [real] radial line, according to which the diffusion of light takes place, there is some breadth, . . . and in the middle of that line there is an imaginary mathematical line to which all the other mathematical lines in that natural line are parallel."[12] So understood, the mathematical ray represents three distinct yet intimately related things: 1) the conceptual limit of the narrowest possible thread of physical light; 2) the shortest possible spatial link between any luminous spot and a corresponding illumined spot in the medium; and 3) a direct measure of light-intensity.[13]

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[11] "Omnis linea, qua pervenit lux a corpore luminoso ad corpus oppositum, est linea naturalis sensibilis, latitudinem quandam habens" (*WP* 2.3, enunciation, p. 63; see also *DA* 4.3.16, 112; *DMS* 2.1, 95; and *PC* 2.18, 169).

[12] "Est ergo in linea radiali, secundum quam fit diffusio luminis, aliqua latitudo, . . . et in medio illius lineae est linea mathematica imaginabilis, cui omnes aliae lineae mathematicae in illa linea naturali equidistantes erunt" (*WP* 2.3, 63 [ll. 59–61]; see references in n. 9, this chapter, and *Paralipomena* 1.8 [*GW*, 2: 21]).

So far, this model reveals a crucial and obvious link between distance (or ray-length) and light-intensity. But the Perspectivists recognized an equally important link between radial obliquity and light-intensity. Pecham, for example, establishes this link by pointing to the fact that the perpendicular ray "is stronger than [all] others, not only because of the absolute nature of the ray, but also because of the mode of incidence on the object." Therefore, he concludes, "the strength of the incident ray varies with the size of the angle of incidence."[14] To be oblique, however, the ray must be so with respect either to illuminated objects or to luminous sources. Let us turn first to the case of obliquity with respect to illuminated objects. Assume that ABDC in Figure 11 represents a segment of a filament of light centered on mathematical ray KL and that it falls perpendicularly upon surface BD. Then rotate this surface around point L so that, for instance, it coincides with ELF and GLH. Since GH is longer than EF, which is in turn longer than BD, the more the surface is rotated (i.e., the more obliquely the ray falls upon it), the larger the excised section becomes. But the amount of light contained by that filament remains constant, so, as it becomes more oblique, its light must be increasingly dispersed to encompass ever larger sections of that surface. The more dispersed the illumination is, however, the weaker its intensity becomes. Hence, the power of the ray must vary continuously with the angle of incidence, the former always decreasing as the latter increases.[15]

The second case of obliquity—with respect to luminous sources—can be understood through Figure 12. Let arc ABC represent part of a luminous

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[13] It is a commonplace in Perspectivist writings that the farther light gets from its source, the weaker it gets (see *DA* 4.3.5 and 4.3.15, 103–104 and 111–12; *WP* 2.22–24, 69–70; and *PC* 1.18, 94. Cf. *Paralipomena* 1.7 [*GW*, 2: 21]). It should be noted that, in relating the decrease of light-intensity to distance from the source, the Perspectivists invariably pointed to the fact that light-rays spread out as they get farther from their source-point. The Perspectivists did not actually provide an explicit statement of the inverse proportionality between distance and intensity in terms of the mathematical ray. On the other hand, as is evident from their mathematical treatment of "pyramids" or cones of radiation, they normally thought of such cones in two-dimensional rather than three-dimensional terms. Thus, by relating light-dispersion along the horizontal to distance from the vertex/source within a *cross-section* of the cone, they implied a simple inverse proportionality between distance and intensity.

[14] "Amplius perpendicularis radius fortior est aliis, non solum propter conditionem radii absolutam, sed propter modum oriendi super rem obiectam . . . Fortitudo igitur radii cadentis est secundum quantitatem anguli quem radius constituit in cadendo" (*PC* 2.6, 160–62 [ll. 79–83]; the English translation in the text is Lindberg's). See also *WP* 2.47, 81 (ll. 60–63); *DA* 1.5.18, 10; *Paralipomena* 4.1 (*GW*, 2: 104 [ll. 32–33]); and *DMS* 2.3, 113.

[15] For a similar sort of analysis of obliquity as the cause of decreased light-intensity, see *Paralipomena* 4.2 (*GW*, 2: 105) and Maurolyco, *Photismi*, pt. 1, theorem 2 (p. 7 in Crew's translation).

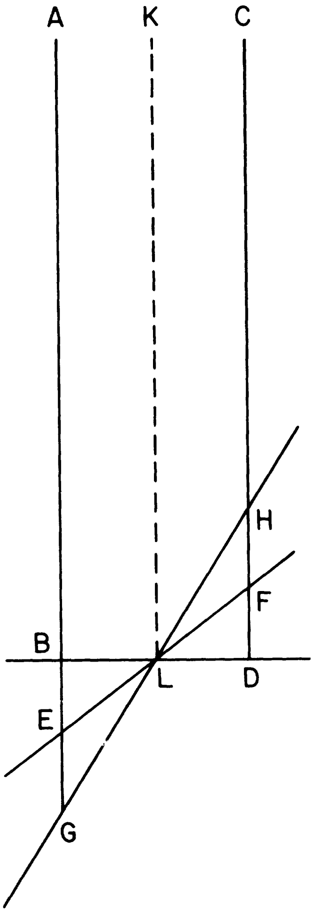

FIGURE 11

surface with center D, and upon that surface let point B represent an arbitrary spot of luminosity. That spot will generate its own sphere of illumination, which can be represented by EFG. Meantime, let HFK represent a sphere of propagation concentric with surface ABC and coincident with EFG at F. Finally, from center D of the luminous object let ray BF be drawn so that it is flanked by the two rays BNL and BOM. Ray BF will therefore be perpendicular to tangent EBG of surface ABC (the luminous source), whereas BNL and BOM will be oblique to it. Obviously, because they are radii of the same sphere EFG and, consequently, of equal length, rays BN, BF, and BO will have the same illuminative power at points N, F, and O. However, when those same rays are extended to points L, F, and M of the luminous object's sphere of propagation HFK, oblique rays BL and BM will be longer than perpendicular ray BF. The more oblique they become—i.e., the greater the angle they form with BF—the longer they will become.

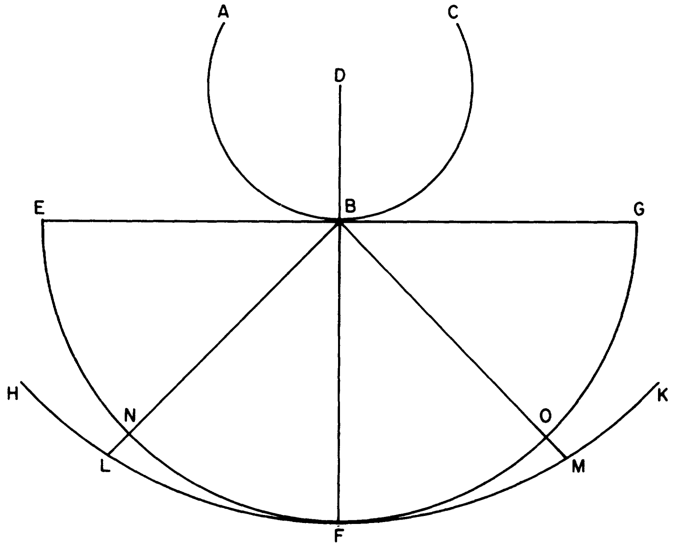

FIGURE 12

And the longer they become, the weaker they become.[16] Hence, in this case the relationship between obliquity and light-intensity is directly mediated by ray-length. It should nevertheless be understood that here, as in the first case, what ultimately causes the variation in light-intensity is dispersal, so the two cases are at least conceptually linked.

Another way in which the Perspectivists explained the relation between obliquity and intensity with respect to the light-source is illustrated in Figure 13. Let ABG represent a luminous surface upon which selected radiative sources A, B, and G lie. Each of these spots will radiate its species hemispherically, the limit of possible radiation being represented by tangents HAT, EBZ, and IGK, since to surpass those tangents would be to radiate into the luminous body itself. Let HLK represent an arc on a given sphere of radiation concentric with the whole luminous source ABG. The problem is to demonstrate that from any one of the points on that surface—B in this case—the ray perpendicular to the tangent through that point will be the strongest. The proof itself centers on the fact that the chosen hemispheres of radiation will overlap and thus reinforce each other, such that, for instance, A's hemisphere HIT will overlap with G's in section ILT to give double reinforcement. Meanwhile, B's hemisphere will add another quantum of reinforcement in that same area. That area of maximum overlap, which is always directly opposite B, is thus the most reinforced possible. The more points we take into account in this analysis, the smaller that area of maximum overlap becomes, until it becomes virtually a point in size.[17]

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[16] Bacon seems to evince a similar line of reasoning in *DMS* 5.1, 234 (ll. 80–87).

[17] See *WP* 2.21, 69, for Witelo's version of this demonstration. For its source see al-Kindi,

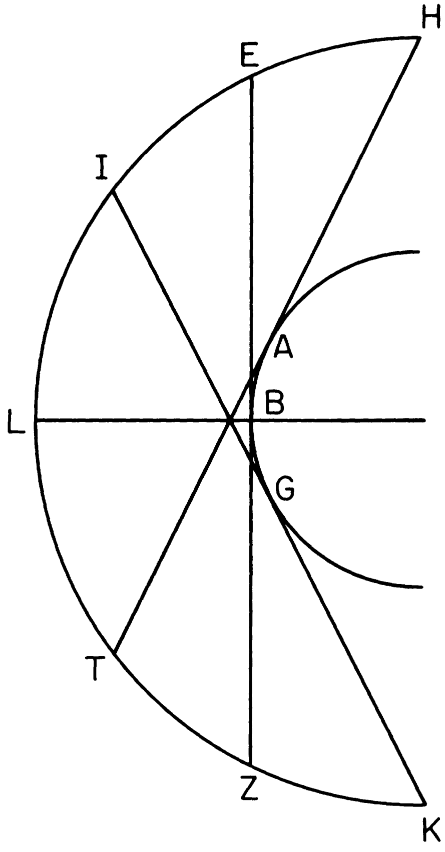

FIGURE 13

There are three important conclusions to be drawn from the Perspectivist account of obliquity. First, rays that are perpendicular to luminous surfaces are the strongest because they represent the shortest point-to-point link between luminous surfaces and the spheres of illumination generated by them. Hence, the ray is truly a radius, as indeed the Latin term *radius* ("ray") connotes. That is why those spheres can be selectively resolved into a locus of "points" connected orthogonally to corresponding "points" of radiation on their generating surfaces. It is also why the Perspectivists regarded the perpendicular ray as the most economical and "natural" line of radiation. Second, rays that fall perpendicular to illuminated surfaces are the most powerful, because their action is concentrated on the smallest possible areas of those surfaces. By so concentrating their action, they act

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De aspectibus, proposition 14 (ed. A. Björnbo, *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, vol. 26, fasc. 3 [Leipzig: Teubner, 1912], 23–25). See *DMS* 5.2, 252, for Bacon's use of the same theorem.

most economically and, therefore, most naturally.[18] Third, granted equality in their length, the power of rays incident on the same point is a function of the angle of incidence. The power of each ray in such a case is therefore inversely proportional to its obliquity, the perpendicular ray being taken as an absolute standard or normal from which both obliquity and power are measured.

The Quantization of Light

It is only natural that, having successfully reduced the physical act of illumination to terms of discrete points, lines, and angles, the Perspectivists should have treated light itself as if it consisted of tiny particles. Witelo, for example, explained light-intensity by recourse to *lux minima*, the smallest effective "unit" of light.[19] A function of compression (to use Witelo's own terminology), intensity increases with the packing or aggregation of such units. Conversely, as it diffuses over greater distances and, consequently, over greater spaces, light will become more scattered and less intense.[20] The Perspectivists' tendency to quantize light was the natural outgrowth of a long-standing use of mechanical analogies to explain the interaction of light and physical bodies. Sight was thus accounted for in terms of physical impressions struck by incident light on the crystalline lens.[21] Reflection became a matter of sudden impact and rebound.[22] And, as we have seen, refraction was understood as a sort of incomplete rebound which merely deflects the light instead of reflecting it fully at the point of impact.[23]

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[18] As we saw above (n. 8, this chapter), the Principle of Natural Economy mandates that all natural actions occur without waste. Applied to light-radiation, this requires two things: that the light follow the shortest spatial path possible, and that it act without wasted effort. Thus, when it acts according to the shortest path, it must also act with maximum efficiency or power. But the shortest path between any luminous point and a point on a given illuminated surface will be perpendicular to that surface or its tangent at that point. Accordingly, the perpendicular must be the strongest and most natural path for light to follow.

[19] "Lux minima dicitur, quae si dividi intelligatur, non habebit amplius actum lucis" (*WP* 2, definition 5, p. 61).

[20] "Lucem compressam fortiorem esse luce disgregata" (*WP* 2, supposition 1, p. 61; see also *DA* 4.3.15, 111–22, and *PC* 2.17, 167–69).

[21] The empirical justification for this stamping imagery (connoted by the continual use of the term *impressio*) lies in the fact that we retain after-images of bright colors or light (see, e.g., *DA* 1.1.1, 1; *WP* 3.6, 87–88; Bacon, *Perspectiva*, pt. 1, chap. 4, distinction 1, pp. 27–28; *DMS* 1.3, 44 [l. 30]–46 [l. 49]; and *PC* 1.1 and 2, 63). For the source of this imagery, see Aristotle, *De anima* 2.12.424a18–23 and *De memoria et reminiscentia* 1.450a.30–450b.12.

[22] *WP* 5.5, 192 and 5.18, 198; *DA* 4.3.17 and 18, 112–13; and *DMS* 2.3, 112 (ll. 91–98); see also *Paralipomena* 1.18 (*GW*, 2: 25). Two obvious sources for the analogy between reflection and physical rebound are Hero of Alexandria, *Catoptrics*, propositions 3 and 4 (*Heronis Alexandrini opera quae supersunt omnia*, vol. 2, fasc. 1, ed. W. Schmidt and L. Nix [Leipzig: Teubner, 1900], 322–38) and Ptolemy, *Optics* 3.19, 98 (ll. 13–21).

[23] "Omne enim corpus diaphanum plus et minus resistit penetrationi lucis, secundum quod est participans diaphanitate plus vel minus. . . . Cum ergo lux pertransiverit corpus aliquod diaphanum oblique, et occurrerit corpori alii diaphano grossiori: tunc corpus grossius resistit luci vehementius, quam prius corpus rarius resistebat: necesse est ergo, quod propter resistentiam illius corporis densioris motus lucis transmutetur: et si resistentia fuerit fortis, tunc motus ille ad partem contrariam refringetur" (*WP* 2.47, 82 [ll. 8–13]; see also *DA* 7.2.8, 241).

In addition to the "dynamic" properties of incident light, the Perspectivists adduced a host of physical attributes for the bodies with which light interacts to account for the results of those interactions. For instance, they thought that opacity depended in equal measures upon the hardness, roughness, and porosity of the body's surface. A perfectly hard, smooth, non-porous surface would be totally reflective. A hard, rough, relatively non-porous surface would be visible but essentially non-reflective, since it would scatter the light that strikes it. A relatively soft surface with labyrinthine porosity would absorb light more or less completely by entrapping it and causing it to reverberate internally where it would dissipate its power beneath the surface. By the same token, finally, a relatively soft surface with regularly arranged pores would be transparent because such pores would allow complete penetration.[24]

These mechanistic explanations strongly suggest a materialist, even an atomist, conception of light and matter. However, it is necessary to emphasize that such "explanations" are analogies only and, consequently, that the implicit atomism upon which they hinge is virtual rather than literal. Light can be quantized conceptually because it acts in certain ways as if it were really composed of physical quanta. In truth, though, the universe remained a plenum for the Perspectivists, a continuum within which all processes were deemed to be absolutely continuous as well. There is no question here of true mechanism in the seventeenth-century sense, for the causal basis of Perspectivists physics was, at bottom, still paramountly formalist, not materialist.[25]

Despite this constraint, the tendency to quantize light had become so prevalent in the Perspectivist account of radiation that it was bound to affect the very theoretical foundations of that account. We have, in fact, already seen how they were subtly altered to allow for discontinuity in the plane of the surface of radiation—that is, since each spot on a luminous surface was supposed to radiate its form everywhere transparency permitted, the resulting sphere of propagation took form as a virtual mosaic of spot-forms. But this virtual discreteness was so far unmatched along the radius of propagation. Radiation outward still happened in what amounts to a sudden, single act of transformation. Thus, the cross-section of a sphere of radiation had to consist of an indefinite number of tiny spots at the circumference linked throughout to the luminous source-spot by continuous radial lines. There is an obvious inconsistency here, for the physical surfaces from which light emanated were held to be actually continuous, and so was the medium through which it emanated. Why, then, posit discontinuity (even if only virtual) in one dimension while yet essentially denying it in the other—and all within the same continuous medium? The answer is that discreteness on the surface of radiation was necessary for the punctiform analysis of visible forms. Discreteness along the actual line of radiation was not.[26] Further, if such discreteness *were* supposed, then the propagation of light would necessarily take place in a spatial succession, which in turn implies that it would take time, and that is tantamount to accepting action at a distance.[27]

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[24] For the various physical properties of bodies that determine how they interact with light, see *WP* 5, definitions 1 and 4, p. 190, and 5.1–3, 192–93; *DA* 4.3.6, 104; *Paralipomena* 1.12, 14, 17, and 22 (*GW*, 2: 22–24 and 31); *DMS* 2.5, 130 (l. 40)–32 (l. 54); and *PC* 2.2 and 10, 159 and 165.

[25] The Scholastic Aristotelians in general, and the Perspectivists in particular, were unanimous in denying that sensation and perception are caused by a flow of matter from the object of sense-perception to the perceiver. For that reason, they were always careful to emphasize that the change that occurred in the transmission of light and color was, at bottom, formal or qualitative in nature (see Aristotle, *De anima* 2.7.418b14–27 and *DMS* 1.3, 44 [ll. 20–29]).

Roger Bacon saw the problem clearly. On the one hand, the idea that light moves in real spatio-temporal succession is extraordinarily useful, because it gives logical force to the mechanical analogies used in Perspectivist optics. On the other hand, action at a distance is as abhorrent to reason as the void within which it must occur is to Nature. The problem is how to justify the one while excluding the other, and Bacon's response was the theory of Multiplication of Species.[28] Bacon describes "multiplication" as a continuous succession of microscopic agent-patient interchanges unfolding over distance—in short, the Aristotelian account of change rendered exceedingly fine. Each spot of *lux* on a given luminous surface is a potential agent "through its active virtue and power."[29] In direct contact with a properly receptive subject or patient (the transparent medium), that spot will inform or transmute each spot of the patient contiguous with it. This transmutation is a function not just of the agent's capacity to act, but also of the recipient's capacity to be acted upon, so that the species "is generated out of the active potentiality of natural or recipient matter."[30] Having assimilated the formal effect or "species" (*lumen*) of the primary agent by its concomitant power of elicitation (*eductio*), each spot in the medium will become an agent in its own right once the effect is actualized in it. As such, it too will transmute the unaffected spots of the medium contiguous with it, and so on *seriatim.* Therefore, "multiplication is an univocal action of the agent and its species, so that light generates light, and generated light generates further light, and so on . . . [such that] the first part of the recipient, having been transmuted and possessing species in actuality, transmutes the second part, and so on."[31]

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[26] In other words, if the action and effects of light on various bodies are to be analyzed according to the ray-model, it is necessary to imagine the surfaces of concentric spheres of radiation as composed of virtually discrete points. On the other hand, if it is assumed that light propagates itself instantaneously, there is no need to imagine the ray itself composed of discrete points along its length. For an excellent discussion of Perspectivist light-theory and punctiform analysis, see Lindberg, *Theories of Vision,* 71–80; see also my "Getting the Big Picture," 578–82.

[27] According to Aristotle (*Physics* 7.2), everything that moves locally must be moved by something else in continual contact with it, so that all locomotion is reduced to four types: pulling, pushing, carrying, and twirling. Any other kind of motion would constitute action at a distance, since the thing moved would be separated from the thing moving it. If light-radiation were a matter of particles actually ejected through space in time, then, once they left the luminous source, those particles would be moving apart from their initial mover. In short, they would have nothing to keep them moving. Moreover, according to *Physics* 4.6–8, these particles could not possibly move through a void, since a void cannot possibly exist, so they must move through a continuous medium. But such media continually resist physical passage, so that the light would decelerate very quickly as it got farther from its source. This, however, is not borne out by experience, which shows that light is transmitted from even the farthest luminous objects (i.e., the stars) in an imperceptible amount of time (see *DMS* 4.3, 222 [ll. 35–57], and *Paralipomena* 1.5 [*GW,* 2: 21]). Nevertheless, the Perspectivists did believe that transparent media posed some resistance to the passage of species (see n. 23, this chapter), but evidently such resistance was not nearly as intense as the physical opposition such media (e.g., air or water) would make to material passage.

[28] The basic theory of species-multiplication goes back to Robert Grosseteste, but it was Bacon who brought it to its final elaboration and who may have been responsible for its diffusion into the general philosophical context of the later Middle Ages. From then on through the Renaissance, the use of "species" and "multiplication" in descriptions of light-transmission was so commonplace as to be unremarkable.

[29] "Tunc enim substantia agentis activa tangens sine medio substantiam patientis potest ex virtute et potentia sua activa transmutare primam partem patientis quam tangit" (*DMS* 1.3, 52 [ll. 151–53]).

Bacon was attempting to justify the physical quantization of light by reinterpreting the original causal account of propagation according to the broadest possible sense of "multiplication." His species pass throughout the intervening medium in a vertical succession of spot-thick spheres whose surfaces themselves comprise a transverse succession of spot-forms. Each line of vertical succession therefore comprises a "ray."[32] Moreover, by contriving his interpretation within the general constraints of Aristotelian causality, Bacon left himself a convenient hedge against the damning consequences of action at a distance. The transmission of light does not entail a passage through empty space, since the medium is only *virtually* discrete. Still, Bacon was quite ready to allow that the species itself, as a purely formal entity, does move in spatio-temporal succession through the medium.[33]

By placing such stress on the successive nature of species-multiplication, Bacon gave his account an unavoidable materialist cast. Granted, the real cause and effect of visible radiation (*lux* and *lumen*) remain absolutely qualitative according to his theory. Still, they are qualities in real motion, and motion must occur in or through something. That "something" is, of course, the continuous transparent medium of Perspectivist physics, but with one critical difference. At Bacon's hands that medium has taken on a distinct internal structure which, even if only virtually imposed, places it in a radically altered light. Considered in terms of contiguity, Bacon's medium provides a matrix within which light-radiation can be understood as a true point-by-point passage. And his medium is not just an inert backdrop for that passage. In its own peculiar way, it is an efficient cause as well, an agent "de potentia activa sua," by means of which it elicits formal effects and passes them on, spot by spot, through space.

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[30] "Ergo non fiet species in potentia receptiva, sed de potentia activa, materie naturalis seu patientis" (*DMS* 1.3, 46 [ll. 62–64]; the English translation in the text is David Lindberg's).

[31] "Et hec multiplicatio est actio univoca agentis et speciei, ut lux generat lucem et lux generata generat aliam, et sic ulterius. . . . quod prima pars patientis transmutata et habens speciem in actu transmutat partem secundam, et secunda tertiam, et sic ulterius" (*DMS* 2.1, 90 [ll. 6–13]; the English translation in the text is Lindberg's). Bacon characterizes light-radiation as an "univocal action" insofar as it is a direct or essential effect of *lux,* whereas, for example, the heating effect of *lux* when it illuminates opaque bodies is "equivocal" insofar as it is accidental to light-radiation (see *DMS* 1.1, 12 [ll. 174–81]).

[32] "Unde pars prima patientis est tanquam centrum commune ad infinitas lineas et radios; sed est terminus omnium intra ad quem continuantur, pars vero agentis est centrum et terminus ad quem contiguantur" (*DMS* 2.1, 92 [ll. 51–54]).

[33] See *DMS* 4.3: "Si vero dicatur quod conceditur, si species corpus esset, quod pertransiret partes spatii in tempore sensibili; et nunc tot sunt generationes singule in singulis partibus quot essent replicationes particulares illius corporis per singulas et easdem partes spatii; ergo sicut replicationes ille corporis moti fierent in tempore sensibili, sic hee generationes in eisdem partibus spatii" (p. 224, ll. 85–90). See also Bacon, *Perspectiva* 1.9.3, 68–71.

It is not surprising, then, that Bacon's analytic focus was so consistently directed to the material substrate in his account of species-multiplication. So clear was his concern in this direction that he insisted emphatically time and again that species themselves are corporeal, not because they are actually bodies, but because they are necessarily *embodied* throughout their passage. "Therefore," Bacon concluded, "since the corporeal medium receiving a species (or, more accurately, from the potentiality of which the species is brought forth . . .) has entirely corporeal being, this species can have only corporeal being." Yet, as he previously warned us, "it is badly and improperly said that the agent dispatches something to the recipient, which flows into it."[34] Light may not yet be matter in real motion for Bacon, but it certainly involves real motion in matter.

Whatever else it does, Bacon's account of species-multiplication shows how closely an essentially static, formalist explanation of light and a kinetic, mechanistic description of its activity have converged in Perspectivist optics. All that separates theory from analogy is the narrow gap between Real and Virtual. The ray, in short, has become virtually equivalent to a trajectory, while light-radiation has become virtually equivalent to temporal passage along it. Hence, according to Witelo:

> we understand a certain kind of movement for light through the medium of resisting bodies, which are more or less capable of [accepting] a luminous impression; [but we adopt this understanding] not because any motion [actually] occurs through a change in place of the light itself, . . . but because light is more compressed or dispersed at the same instant on account of the difference [in density] of the media. This [alteration] we call "motion of this light" here.[35]

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[34] For the first quotation, see *DMS* 3.2, 188 (ll. 33–36); for the second, *DMS* 1.3, 44 (ll. 22–23). The English translation in both cases is Lindberg's.

[35] "Intelligimus quendam modum motionis luminis per medium corporum resistentium, quae secundum plus et minus capacia sunt impressionis luminaris, non quod in transmutatione locali ipsius luminis fit aliquis motus, . . . sed quia lumen in eodem instanti secundum diversitatem mediorum se plus comprimit vel diffundit: et hoc vocamus hic motum ipsius lucis" (*WP* 2.47, 82 [ll. 2–6]). This idea of light being a sort of motion is naturally implied by the use of mechanical analogies in describing the action of light in reflection and refraction, and it is reinforced by the use throughout of words with expressly kinetic connotations—e.g., *motus, transitus,* and *velocius,* this last one being especially significant, since it conveys the idea of speed (see, e.g., *DA* 7.2.8, 240–42; *DMS* 2.3, 111–16; and *PC* 1.15, 91). For Witelo and Pecham, both of whom accept the instantaneous transmission of light, such language is obviously figurative (see also *Paralipomena* 1.5 [*GW*, 2: 21]).

More important, given its kinetic and dynamic characteristics, the ray has become equivalent to a specific kind of trajectory: one traced by a projectile shot directly outward from its natural place. In other words, the Perspectivist physics of light can be understood through direct comparison with the physics of natural motion or free-fall. The rectilinearity of the ray is therefore a function of the naturalness of the course that light follows in its projection.[36] Because that "projection" is exactly contrary to free-fall, though, the kinetics and dynamics of light ought to be precisely contrary to those of a falling body. Instead of gaining power and speed like a falling body, light will continually lose power and speed as it follows its natural course. Like a falling body, however, light will act as efficiently as possible—that is, with maximum speed and power—when it follows that course.[37] In a sense, then, the ray is simply a mirror-image of the trajectory of free-fall, a line of natural ascent rather than one of natural descent.

Comparison with Descartes's Theory of Light

At this juncture several points can be raised about the relationship between Descartes and the Perspectivists. First, there is no question that the Perspectivist and Cartesian theories of light are polar opposites in one critical respect. Whereas the Perspectivist theory is utterly dependent upon the formal causation of *lux* and *lumen.* Descartes's is consciously and artfully designed to preclude just such causation. Indeed, Descartes makes no bones about his wish to deliver us " from all those tiny images fluttering through the air, called *intentional species,* which exercise the imagination of the Philosophers so much."[38] Yet we should not allow this one obvious (and epistemologically crucial) difference to obscure the many fundamental similarities between the two.

In both theories, light-radiation is basically an exercise of power, generated at a luminous surface and immediately actuated in a transparent medium. In both, the physical exercise of that power is doubly determined: by ray-length and obliquity for the Perspectivists, by horizontal and vertical displacements for Descartes.[39] In both theories, the ray is a mathematical fiction that represents a perfectly efficient line of "mean" light distribution.[40] In both, moreover, the shape and direction of radiation are at bottom a matter of causal priority—of *lux* to *lumen* in the Perspectivist theory and of centrifugal nudge to pressure transmission in the Cartesian theory—not of spatio-temporal succession.[41] In both theories, as well, the medium is essentially continuous, as is the act of radiation. Both theories nevertheless reflect clear recognition of the need to render the medium somehow discontinuous in order to allow for a discrete radial analysis of propagation. This the Perspectivists did by assuming that the medium, though really continuous, is nonetheless virtually discontinuous, which thus permitted them to treat light-radiation as if it were spatially successive. Descartes took a subtly different tack, positing actual discontinuity within a virtual continuum. Either way the result was the same; both the Perspectivists and Descartes could legitimately treat light atomistically and mechanistically without claiming it to be matter actually in motion. That way both avoided the snare of action at a distance. Perhaps the most telling similarity between the two, however, lies in their equally heavy reliance upon the distinction between Real and Virtual, since it was through this all-important distinction that both ultimately justified equating the physical action of light with that of a projectile hurtling through space.

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[36] That is, since a straight-line projection is both shortest and most powerful, it must be most natural.

[37] "Omnes enim motus naturales, qui fiunt secundum lineas perpendiculares, sunt fortiores, quoniam coadiuvantur virtute universali coelesti secundum lineam rectam brevissimam" (*WP* 2.47, 81 [ll. 58–59]). For similar uses of the free-fall (Natural Motion) analogy with light, see *Paralipomena* 1.4 (*GW,* 2: 20–21); *DMS* 2.3, 111 and 5.1, 235; and *PC* 2.6, 161.

[38] "Et par ce moyen [i.e., thinking of light and color as mechanically transmitted] vostre esprit sera deliuré de toutes ces petites images voltigeantes par l'air, nommées des *especes intentionelles,* qui trauaillent tant l'imagination des Philosophes" (*AT,* 6: 85 [ll. 24–27]).

[39] The Perspectivist theory of light implies a notion of "determination" insofar as light has an absolute power or intensity according to its radial distance from the source. When the light strikes a physical body, however, that absolute power is diminished in proportion to the angle at which the light strikes. Thus, the actual intensity of the light upon striking is determined by *both* its absolute power and its relative loss of power according to the obliquity of incidence.

So, for all its seeming novelty, Descartes's theory emerges as a natural continuation of the Perspectivist effort to narrow the gap between what remained an essentially static *explanation* of light and a kinetic *description* of its physical activity. Viewed from this standpoint, Descartes's real contribution to light-theory lay in his attempt to make the Perspectivist account simpler and more logically consistent by literalizing its mechanistic implications. Consequently, he channeled his efforts in three basic directions. First, he strengthened the Perspectivist emphasis on the role of the material medium in the transmission of light. Second, he added logical force to the Perspectivist conception of radiation as a pseudo-kinetic act along a perfectly straight (and wholly imaginary) trajectory, while preserving the overall continuity of both the act and the medium in which it unfolds. And finally, he streamlined the physics of light by obviating the need for formal causation. As radical as it may at first seem, this step amounts to little more than substituting mechanical pressure for formal change within the same static, plenistic system. On a purely physical level, then, Descartes's theory constitutes a logical restructuring rather than a fundamental destruction of its Perspectivist antecedent. This is not to minimize the changes that Descartes wrought, especially at an ontological and epistemological level, where the implications of his new theory were radical in the extreme.

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[40] "Possumusque," says Descartes in a letter of March 1638, "singulos ordines horum globulorum [i.e., of Air] ab obiectis ad oculos protendentes (saltem si philosophice loqui libet) *radios materiales* appellare, ad distinctionem *radiorum formalium*, qui secundum lineas Mathematice rectas atque indiuisibiles ferri intelliguntur" (to Ciermans, *AT*, 2: 77 [ll. 2–6]—my emphasis).

[41] See chap. 3, n. 45.

V. THE PERSPECTIVIST ANALYSIS OF REFRACTION

By the late thirteenth century the Perspectivists had laid the groundwork for a fairly coherent mechanistic account of light based on the analogy between radiation and projectile motion. Light had accordingly been transformed by them into a projectile, the ray into a natural trajectory, and illumination into an impact phenomenon. Light-intensity—which the Perspectivists took as an *explicit* function of ray-length—had thus become an *implicit* function of radiative speed and angle of impact. And radiative speed in its turn had become an implicit function of ray-length taken as a measure of distance. The ulterior purpose of this mechanistic account was to provide a simple, rational way of understanding the interaction between light and physical objects. Since the Perspectivists viewed refraction as a particular case of such interaction, they quite naturally tried to make sense of it in mechanistic terms. As we shall now see in detail, they carried out that attempt by transforming refraction into an impact phenomenon wholly determined and explained by the dynamic interaction of projected light and transparent media. We shall also see that the crux of this transformation lay in the implicit relationship between intensity, as a function of the power of impact, and speed of propagation.

The Physical Model

That the fifth book of Ptolemy's *Optics* was the immediate source for the Perspectivist understanding of refraction is beyond doubt. Witelo, for example, copied Ptolemy's tables of refraction almost *verbatim,* and Witelo's version provided the standard measures of incidence and refraction for the next four centuries.[1] But Ptolemy's influence on the Perspectivists was by no means limited to those measures. His sketchy "explanation" of refraction by mechanical analogy served as a basic model for the Perspectivist (and thence the Cartesian) analysis of refraction. The specific analogy Ptolemy offered rested on the assumption that the optical properties of various transparent media—air, water, and glass in his case—were somehow similar to, and perhaps even linked to, their physical properties. Accordingly, these media could be likened to fluids of a given rarity or density, and light could be thought of by extension as a projectile moving through them. When passing from a rarer to a denser medium, such a projectile would be impeded in its progress and would therefore lose speed and power. Because that loss would be contingent on the density of the new medium, it follows that the greater the one, the greater the other in direct proportion. So too, it follows that the greater the loss of speed and power, the greater the deflection toward the normal.

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[1] Ptolemy's three refraction tables (air to water, air to glass, and water to glass) are listed respectively in *Optics* 5.11, 5.18 and 5.21, 229–30, 233–34, and 236–37. With one puzzling exception—the first value for the angle of refraction in the air-water table, which he changed from 8 degrees to 7.75 degrees—Witelo reproduced Ptolemy's tables exactly in *WP* 10.8, 412. Those tables, mediated by Witelo, were known to and used by Descartes (*AT,* 11: 645–46) and Kepler (*Paralipomena* 4.6 [*GW,* 2: 109]).

As outlined by Ptolemy, this model has a number of crucial implications, some of which have already been pointed out in the discussion of Descartes's physical analysis but which nonetheless bear repeating. First, if we assume that the projectile's loss of speed is directly related to the density of the medium it strikes, we can logically conclude that the speed of such a projectile through any medium should be inversely proportional to the medium's density.[2] In short, the relationship between density and speed should be mathematically precise. Yet we have also related speed and deflection in claiming that the slower the projectile, the more it inclines toward the normal. We can thus reasonably conclude that there ought to be some exact mathematical relationship between speed and angular deviation as well as between density and angular deviation.[3] Likewise, we can reasonably suppose that, when the same projectile passes through the same interface at various angles, the resulting deviations will somehow bear a constant relationship both to each other and to their generating angles of incidence.[4] Finally, it makes sense to relate refraction directly to reflection, since reflection seems to be an extreme case of refraction in which the density-differential between the two media is too great to allow traversal at the interface.[5]

There is no way of knowing whether Ptolemy actually applied this model and its attendant implications to an explicit dynamic explanation of refraction.[6] We do know, however, that it played a central role in the Perspectivist attempt to achieve such an explanation. To understand how, we need only look at Alhazen's *De aspectibus,* since that was the primary avenue through which Ptolemy's model was passed to the rest of the Perspectivists. Equally important, that model was passed on by Alhazen in a particular, somewhat altered, form.

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[2] See Aristotle, *Physics* 4.8.215a28–215b12, for the primary source of this assumption. The line of reasoning is this: assume that the light passes from one medium into another that is twice as dense and, therefore, that it loses half its speed or power in proportion to the density-differential. It follows then that its speed in the denser medium will be half what it was in the rarer. Likewise, if it were to pass into a medium that is three times as dense, the light would lose two-thirds of its original speed and would thus move with a third of its original velocity in the denser medium.

[3] See, e.g., *PC* 1.15, 89–90 (ll. 385–89).

[4] This, in fact, seems to have been Ptolemy's initial hypothesis (later somewhat modified), which was based on the assumption that the angles of incidence and refraction would be constantly proportional (see my "Ptolemy's Search," 230–35). Much the same sort of assumption seems to underly Grosseteste's "half-angle law" of refraction in the *De iride* (see Bruce S. Eastwood, "Grosseteste's 'Quantitative' Law of Refraction: A Chapter in the History of Non-Experimental Science," *Journal of the History of Ideas* 28 [1967]: 403–14). And it also seems to provide the basis for Maurolyco's claim of constant proportionality between the angles of incidence and deviation in *Photismi,* pt. 2, bk. 1, theorem 10 (pp. 57–58 in Crew's translation).

[5] See chap. 2, nn. 22 and 28, and chap. 3, n. 23.

In the first place, Alhazen amplified Ptolemy's analogy in certain important ways, by providing some key illustrative cases that are lacking in Ptolemy's account. One clear instance is Alhazen's discussion of what happens when we hurl an iron ball at a thin plank:

> If one takes a thin board and fastens it over a wide opening, and if he stands opposite the board and throws an iron ball at it forcefully and observes that the ball moves along the perpendicular to the surface of the board, the board will yield to the ball; or, if the board is thin and the force moving the ball is powerful, the board will be broken [by the ball]. And if he [then] stands in a position oblique with respect to the board and at the same distance as before and throws the ball at the same board, the ball will be deflected by the board (unless the latter should be excessively delicate) and will no longer be moved in its original direction, but will deviate toward some other direction.[7]

By the same token, adds Alhazen, if we strike a log along the perpendicular with a sword, it will cut into the log some given distance, whereas the more oblique we make the blow, the shallower the cut until, finally, the sword will simply glance off the surface.[8]

These examples illustrate two important points. First, the more direct the blow is, the more strongly it will penetrate, which is to say (as indeed Alhazen does emphatically) that a perfectly direct blow along the perpendicular is the most effective of all. Second, the more obliquely the blow is delivered, the more weakly it will penetrate and the more its subsequent path will deviate from the initial line of impact. The deviation itself will be toward the normal when the passage is from a rarer into a denser medium (e.g., from air into wood) and away from the normal under the opposite circumstances.[9]

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[6] The only extant version of Ptolemy's *Optics* is the one that was available in the West through Admiral Eugenius of Sicily's twelfth-century Latin translation from the Arabic. Nothing in either Arabic or Greek survives. According to Eugenius, the two Arabic texts available to him were incomplete, and his claim is borne out by the abrupt way the fifth and last book concludes. Thus, the lack of any dynamic explanation of refraction in the *Optics* as we now have it does not necessarily mean that Ptolemy did not supply one in the complete original text.

[7] "Si enim aliquis acceperit tabulam subtilem, et paxillaverit illam super aliquod foramen amplum, et steterit in oppositione tabulae, et acceperit pilam ferream, et eiecerit eam super tabulam fortiter, et observaverit, ut motus pilae sit super perpendicularem super superficiem tabulae: tunc tabula cedet pilae aut frangetur, si tabula subtilis fuerit, et vis, qua sphaera movetur, fuerit fortis. Et si steterit in parte obliqua ab oppositione tabulae, et in illa eadem distantia, in qua prius erat, et eiecerit pilam super tabulam illam eandem, in quam prius eiecerat: tunc sphaera labetur de tabula, si tabula non fuerit valde subtilis, nec movebitur ad illam partem, ad quam primo movebatur, sed declinabit ad aliquam partem aliam" (*DA* 7.2.8, 241 [ll. 13–20]—English translation from Lindberg, "The Cause of Refraction," 26).

[8] "Et similiter, si acceperit ensem, et posuerit coram se lignum, et percusserit cum ense, ita ut ensis sit perpendicularis super superficiem ligni: tunc lignum secabitur magis: et si fuerit obliquus, et percusserit oblique lignum: tunc lignum non secabitur omnino, sed forte secabitur in parte, aut forte ensis errabit deviando: et quanto magis fuerit ensis obliquus, tanto minus aget in lignum" (*DA* 7.2.8, 241 [ll. 20–24]). See *DMS* 2.3, 110 (l. 81)–12 (l. 98), for an echo of this example.

To explain these observations dynamically seems easy enough. The ball or sword has lost speed and power after coming into contact with the impediment, because the plank or log has posed some counter-force to the initial force of penetration. When it is weaker than the force of penetration, this counter-force is overcome, but when it is too powerful to be overcome, it repels the blow and causes the ball or sword to reflect. To get a firmer grasp of the situation, though, Alhazen imports into the analysis a familiar device. He resolves the overall dynamic situation into horizontal and vertical components:

> Now if the motion within an object is oblique with respect to the surface of the object, it is composed of a motion in the direction of perpendicular passage . . . and of a motion along the perpendicular to this [first] perpendicular. Therefore when light is moved through a dense transparent substance along an oblique line, its passage through that transparent substance takes place by means of a motion composed of the two aforesaid motions.[10]

Such a resolution implies that the dynamic situation is twofold rather than simple, because it is obvious that there is a loss of speed and power in *both* the horizontal and vertical directions. That explains not only why the speed and power of the ball or sword are diminished along the vertical after penetration, but also why the ball or sword is deflected away from its original path toward the normal. The deflection must be a function of resistance along the horizontal, so that the more obliquely the ball or sword strikes the surface the more obvious the effect of that resistance will be.[11]

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[9] Just as refraction toward the normal is explained in terms of a loss in the projectile's speed and power when it strikes the interface of a denser medium, so refraction away from the normal is explained by a commensurate gain in its speed and power when it passes into a rarer medium (see *DA* 7.2.8, 242 [ll. 1–13], and *WP* 2.47, 83 [ll. 10–19]).

[10] "Et motus in corpore, in quod transit, si fuerit obliquus super superficiem illius corporis, componitur ex motu in parte perpendicularis transeuntis in corpus, in quo est motus, et ex motu in parte lineae, quae est perpendicularis super perpendicularem, quae transit in ipsum. Cum ergo lux fuerit mota in corpore diaphano grosso super lineam obliquam: tunc transitus eius in illo corpore diaphano erit per motum compositum ex duobus praedictis motibus" (*DA* 7.2.8, 241 [ll. 33–38]—English translation from Lindberg, "The Cause of Refraction," 28. See also *DA* 4.3.18, 112–13; *WP* 2.47, 82 [ll. 61–64]; and *Paralipomena* 4.2 [*GW*, 2: 84 (ll. 20–24)]).

[11] Both Alhazen and Witelo claim that the essential resistance of any medium to passage occurs along the horizontal, so that, on the one hand, when a projectile (or light) passes into a rarer medium, it gains speed while diverging from the normal on account of a lifting or lessening of that resistance along the horizontal. On the other hand, when it passes into a denser medium, it loses speed while verging commensurately toward the normal on account of an increase in horizontal resistance (see references in n. 9, this chapter). Empirically we know that the more obliquely the projectile traverses either interface, the more noticeably it is deflected. Hence, it follows logically that the more obliquely it strikes the interface, the more the resistance along the horizontal is a factor.

However, as Alhazen was at some pains to emphasize earlier in his account, "motion along the perpendicular is stronger and easier; and among oblique motions that which is nearer the perpendicular is stronger and easier."[12] So what deflection toward the normal *really* accomplishes is to channel the ball or sword into a stronger or "easier" path than it would otherwise have were it to continue along the original line of motion after suffering its natural loss of speed and power at the surface.

Five critical points emerge from Alhazen's account as just outlined. First, his resolution of motion into horizontal and vertical components gives a very clear precedent for Descartes's use of the same analytic device. Indeed, there is little reason to doubt that Descartes borrowed it either immediately from Alhazen or mediately through Witelo or Kepler.[13] Second, the ball-and-plank example Alhazen adduced is remarkably similar to the tennis-ball-and-net example that Descartes employed. Whether Descartes actually borrowed this particular example and then modified it is impossible to prove, but the similarity is both obvious and suggestive. Third, what gives Alhazen's ball-and-plank analogy crucial ulterior significance is that it suggests that refraction is a surface phenomenon. If so, then *the dynamic effect of the resistive medium's density is concentrated entirely at its surface.* This in turn implies that the dynamic change that causes refraction is instantaneous rather than continuous.[14] Fourth, like Descartes after him, Alhazen explicitly linked reflection to refraction on the basis of both resistance to penetration (e.g., when the ball strikes perpendicularly and yet rebounds) and insufficient forward momentum of the ball when its angle of incidence is too oblique. Fifth, by pointing out that when it is deflected toward the normal the projectile assumes a more powerful or easier path, Alhazen imported a new, and profoundly important element into the account. In fact, as will become evident, this analytic element formed the very heart of the Perspectivist theory of refraction.

The Physical "Explanation"

For the Perspectivists, as for Descartes, the basic model of projection and collision provided an analogical rather than a truly explanatory basis for understanding light-refraction. Light was indeed assumed by the Perspectivists to act like, but not in reality to be, a projectile. Moreover, the Perspectivists recognized that light acted like a projectile only up to a point. Unlike a real projectile, for instance, light was not actually material, nor did it necessarily take time to propagate according to the Perspectivists. Also, unlike a real projectile passing from air to water, a "quantum" of light propagated radially did not seem gradually to assume a perpendicular trajectory after slamming into the denser medium's surface at an angle.[15] The change in the direction of radiation appeared to be both sudden and complete at the point of impact—hence, the use of the ball-and-plank example in which the second medium was reduced to a virtual surface.

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[12] "Patet quod motus super perpendicularem est fortior et facilior: et quod de obliquis motibus ille, qui vicinior est perpendiculari, est facilior remotiore" (*DA* 7.2.8, 241 [ll. 24–26]). Note Alhazen's use of *facilior* ("easier") and its similar use by Descartes (see chap. 3, n. 38).

[13] See Lindberg, "The Cause of Refraction," 34–35.

[14] The instantaneity of dynamic change at the surface of refraction was, of course, crucial to Descartes's analysis. For explicit claims by the Perspectivists that refraction is a surface phenomenon only, see *DA* 7.2.4, 235; *WP* 2.43, 77–78; *PC* 3.2, 212; and *Paralipomena* 1.12 (*GW*, 2: 22). Since by definition a surface has no depth whatever, its traversal must take no time.

In certain critical ways, though, the collision model appeared to be perfectly suited to the facts as the Perspectivists understood them. Optical density did seem to be related to physical density. After all, light refracted when it passed from air into water, as it did when passing from water into glass; so there seemed to be an evident parallel in gradations between optical and physical density among these three transparent media.[16] In addition, since transparent media were perforce corporeal, and since light could only pass through transparent media, there seemed to be a direct link between the continuous decrease in intensity or power of light over distance and the physical resistance of the medium through which it passed.[17] By analogy with a projectile, the loss of the light's power or intensity seemed virtually equivalent to a loss in speed. Hence, as far as the Perspectivists were concerned, the more optically dense the medium, the more weakly or slowly light passed through it and, as a result, the less intense its action was in it. That, according to the Perspectivists, was why light appeared weaker or murkier in a denser medium, such as water, than it did in a rarer medium, such as air.[18]

Refraction thus seemed to involve two distinct, but somehow dynamically related, phases for the Perspectivists. The first of these phases occurred at the surface of the denser medium, where the light underwent an abrupt jolt of resistance that caused it to slow down and deviate from its original path toward the normal. The subsequent phase occurred within the medium, where the light encountered a natural and continuous internal resistance that caused it to lose power or speed continuously. The basic factor in both cases was the same: the density of the medium. Yet both were quite different in kind, the one being sudden or violent and the other being continuous or natural. This implicit division of the two phases of refraction into "violent" and "natural" therefore depended on discontinuity or continuity of density. Wherever there was discontinuity, the light would suffer an unnatural diminution in its forward momentum.

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[15] Oddly enough, toward the end of the very proposition cited above (n. 14, this chapter), Pecham raises the possibility that light may indeed tend to diverge from a rectilinear path as it passes through a continuous medium. The acceptance of this same possibility may well underly Witelo's peculiar assertion (*WP* 2.35) that the farther they diverge from their source-point, the more light-rays approach parallelism ("Radii ab uno puncto luminosi corporis procedentes, secundum linearum longitudinem ad aequidistantiam sensibilem plus accedunt"—enunciation, p. 73).

[16] Witelo expands this air-water-glass series to include a spectrum of transparent substances ranging in order of density/refractivity from aither, through fire, air, and various liquids to a selection of crystals (see *WP* 10, preface, pp. 404–405).

[17] See *DA* 7.2.8, 241–42; *WP* 2.47, 81–82; and *DMS* 4.1, 211.

[18] See *DA* 7.7.38, 270–71, and *WP* 10.10, 414.

To an extent, the dynamics of refraction as understood by the Perspectivists can be properly interpreted in terms of violence or unnaturalness. But if we look at the dynamic change from the standpoint not of power and speed, but of direction, it is clear that what happens at the interface between two media need not be unnatural at all. When light strikes such an interface orthogonally, it passes through unrefracted and therefore maintains a perfectly rectilinear trajectory, which is in fact the most natural and efficient trajectory possible.[19] It is, after all, along such a trajectory that light acts most powerfully. Moreover, since a perpendicular trajectory is the easiest or least impeded, the more obliquely the light falls upon the surface of a denser medium, the more impeded, inefficient, and unnatural its passage must be.[20] Accordingly, when light strikes the surface of a denser medium at an angle, it meets two distinct resistances: a "natural" resistance perpendicular to the surface of the medium (i.e., along the vertical), and a "violent" one parallel to that surface (i.e., along the horizontal). When light strikes the medium's surface orthogonally, only the first of these resistances is in play, so the passage is minimally impeded or disturbed. As the angle of incidence increases, however, the second resistance comes increasingly to the fore and pushes the light more and more along the horizontal toward the normal.

The Final Cause

So far we have located the ultimate physical cause of refraction in this horizontal resistance, but for the Perspectivists there was an even deeper metaphysical cause governing refraction. Remember, first, that the Perspectivists thought the degree to which light was deflected toward the normal depended upon both the density of the medium and the angle at which the light struck the medium's surface. Remember, too, that in assuming such a deflected trajectory, the light was supposed by the Perspectivists to follow a more powerful course, since its power increased with the inclination toward the normal. Since the most powerful course it could take was along the normal itself, that trajectory was the most natural, efficient, and easiest one possible. Therefore, in being deflected horizontally, the light assumed an easier path through the medium. As Witelo put it:

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[19] "Quapropter cum natura eligit rectam lineam nisi impediatur fortius, aget per species venientes super lineas rectas; . . . Ceterum, linea recta est uniformis et equalis; sed in omnibus aliis est difformitas et diversitas et quedam inequalitas. . . . Et dicitur *Libro de causis* quod omnis virtus unita est fortioris operationis; sed diversitas et difformitas contrariantur unitati et unitioni et uniformitati. Quapropter cum secundum lineam rectam fit uniformitas virtutis in suo decursu, . . . manifestum est quod fortior est virtus veniens super rectam lineam quam super alias" (*DMS* 5.1, 232 [ll. 47–61]).

[20] This is to say that, if the light were to traverse the interface and were to continue in a straight line with its path of incidence, that passage would be inefficient and unnatural. Hence, the more oblique that hypothetical passage, the more difficult, inefficient, and unnatural it would be.

> When light strikes a denser transparent body obliquely, it extends [through it] along a straight line that approaches [the normal], since the easiest motion is along the perpendicular. Hence, it happens that it will incline toward whatever direction easier passage lies rather than toward that direction it would have followed [had it continued] along the line of incidence.[21]

This means that the horizontal or "violent" resistance of the medium was actually thought by the Perspectivists to channel the light into an easier, more efficient, and more natural path than it would have followed had it continued through the interface along the original line of incidence. For, by being deflected at the interface, the light will have regained some of the power and speed lost at the moment of impact when it encountered both the vertical and horizontal resistance.

This point can be made clearer by recourse to Figure 14. Assume that the light strikes interface FD along ray AB according to angle of obliquity ABD. The power with which it strikes that interface will be a function of two things: its forward speed (which varies with distance from the source) and its obliquity. Meanwhile, the medium itself will oppose such penetration according to two things: its resistance along the vertical and its resistance along the horizontal. Now if the light were to strike the interface at point B and then continue in a straight line along BC′, it would suffer an overall loss of power and speed due to *both* of these factors. On the one hand, some of its original power and speed will have been lost through the medium's resistance to vertical passage. On the other hand, an additional loss in power and speed will have been occasioned by the medium's resistance along the horizontal. But instead of continuing along BC′, the light is actually diverted to BC, which inclines toward the normal. The action of the light at point C will thus be commensurately stronger than that at point C′, since ray BC will be less oblique than ray BC′ (angle BCE being less acute than angle BC′E′). So the overall loss in power and speed upon impact will be diminished by the gain in power due to the change in obliquity.[22]

Now, if that gain were just enough to offset the loss suffered at impact by the horizontal resistance of the medium along the interface, then the final loss would be along the vertical only; and that loss is perfectly natural because it is the smallest possible. It follows, therefore, that any loss beyond

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[21] "Cum itaque corpori diaphano grossiori lux incidit oblique, extenditur secundum lineam rectam approximantem ad perpendicularem, exeuntem a puncto, in quo lux occurrit superficiei corporis diaphani grossi, productam super superficiem corporis grossioris, ideo, quia facilimus motuum est secundum lineam perpendicularem. . . . Accidit ergo ut declinet ad partem aliquam, per quam facilior sit transitus, quam per illam partem, ad quam per lineam incidentie movebatur" (*WP* 2.47, 82 [ll. 19–25]).

[22] See *DA* 7.2.8, 241 (ll. 45–54); *WP* 2.47, 82 (ll. 40–49); *DMS* 2.3, 114 (ll. 132–42); *PC* 1.15, 90 (ll. 391–98); and *Paralipomena* 4.2 (*GW* 2: 84 [ll. 17–20]). Bacon explains the deflection of the light into a stronger, easier path in terms of a natural choice ("virtus naturalis generans ipsam speciem appetit faciliorem transitum et eligit illum"), a position that Pecham explicitly rejects ("Nec intelligendum radium ad situm fortiorem declinare quasi per electionem"). Kepler, however, ridicules the entire notion that deflection toward the normal represents a compensation of any kind.

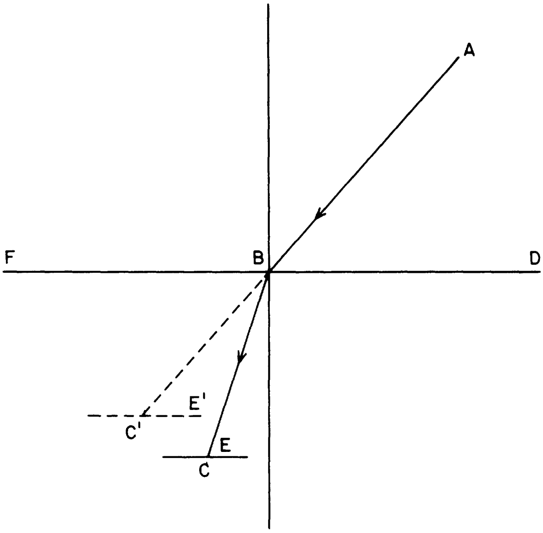

FIGURE 14

this would be wasteful or inefficient, for in that case the light would have expended more effort than necessary to achieve its end.[23] But if we assume with the Perspectivists that Nature does nothing in vain, it follows that such an unnecessary expenditure of effort is impossible. Otherwise, refraction would not be a natural act. Given that it is, we can only conclude that it is as efficient as possible along the vertical. According to Roger Bacon, then: "when a species passes from a subtler into a coarser substance, it maintains its ease of traversal in the second substance, *so that its passage through the two substances is, insofar as possible, proportional and uniform.*"[24]

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[23] Virtually all the Perspectivists believed that light underwent some loss of power when it passed perpendicularly into a denser medium, the loss itself being somehow proportional to the density-differential (see, e.g., *DA* 7.2.8, 242 [ll. 3–8]; *WP* 2.47, 82 [ll. 6–14]; and *DMS* 2.3, 112 [ll. 113–19]). Still, perpendicular penetration was assumed by them to represent the most "natural" or efficient case possible. Accordingly, when the light passes orthogonally from one given medium to another, its force (A) before penetration will be related to its force (B) after penetration in a constant ratio A:B. Therefore, in *every* traversal of that interface, no matter the angle of incidence, the initial force (A') must bear exactly the same relation to the force after penetration (B') as A:B. But the more oblique the penetration, the more the horizontal resistance acts to impede the light and, consequently, the more B' is diminished with respect to A'. Thus, by compensating for just that amount of diminution, deflection toward the normal will restore the relation A':B' to ideal proportionality with A:B.

[24] "Sicut quando species venit a subtiliori in grossius continuat in grossiori facilitatem transitus in corpore secundo, *ut eius incessus in utroque corpore sit proportionalis quantum potest et uniformis*" (*DMS* 2.3, 116 [ll. 163–66]—Lindberg's translation, my emphasis). See also *WP* 2.47, 82 (ll. 19–27); *PC* 1.15, 90 (ll. 387–93); and *Paralipomena* 4.2 (*GW*, 2: 84 [ll. 20–28]), H. J. J. Winter, "The Optical Researches of Ibn al-Haitham," *Centaurus* 3 (1954): 190–210, claims that Alhazen used this principle of vertical conservation, although in fact Alhazen never stated it explicitly (cf. Sabra, *Theories of Light*, 97, n. 3). On the other hand, as Lindberg suggests, such a principle is implicit in Alhazen's explanation of refraction ("The Cause of Refraction," 28; see *DA* 7.2.8, 241 [ll. 45–54]).

Refraction is thus governed by a principle of conservation of power that restores the act as a whole to natural uniformity. Such natural uniformity, however, is a function of ease of traversal, which is itself a function of power along the vertical. Therefore, if the act is to remain natural, power must be conserved throughout along the vertical or normal.

This principle or rule of vertical conservation puts the analysis of refraction on a radically new footing. Until now, the natural uniformity of light-radiation has been understood as a function of the ray's perfect spatial brevity—that is, light has been taken to act naturally insofar as it completes its action within the shortest possible distance.[25] In refraction, however, it is ease of traversal, not spatial brevity, which governs the natural economy of the act. And the need to preserve that ease dictates precisely how much the light must be deflected upon impact in order to cancel out whatever loss in power might be created by the horizontal resistance. As we shall see in the next chapter, this governing rule gives us a simple way of determining what that deflection must be in every case and, therefore, of drawing an exact general relationship between the angles of incidence and refraction.

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[25] See Appendix D for elaboration on this point.

VI. THE PERSPECTIVIST GROUNDS OF THE CARTESIAN PROOF

Ironically, by developing the mechanistic analysis of refraction just outlined, the Perspectivists were on the verge of finding the sine-law. That they never crossed this verge was due in part to their failure to make the analysis more mathematically explicit and in part to their failure to follow the logical imperatives of that analysis to a natural conclusion. So our purpose in this chapter will be to tie the ends that they left dangling by making key tenets of that analysis mathematically explicit and, on that basis, pursuing the analysis to its logical end. In the process, we will be led first to a law of cosines. However, this law raises certain clear difficulties whose resolution demands a slightly revised line of reasoning. As we follow this revised line of reasoning, we will see that it not only parallels the one outlined in the *Dioptrique* but also leads just as ineluctably to the sine-law.

The Mathematical Implications

On the whole, despite a free use of geometrical reasoning, the Perspectivist account of refraction is mathematically vague and suggestive rather than exact. The apparently straightforward claim that light-intensity varies with distance illustrates this point. Evidently, the Perspectivists meant that light-intensity bears an *exact* inverse proportionality to distance; to double the one is thus to halve the other, and so on. But the Perspectivists themselves never expressed the relation in such a precise and explicit way. We can only infer, albeit quite reasonably, that this is how they actually understood it.[1] The same holds for a variety of what we now take to be precise mathematical laws or rules in Perspectivist optics. Their very form as exact mathematical statements is a matter of logical inference, not of explicit symbolic formulation. With this in mind, let us examine certain of those implicit laws with the intent of giving them as suitably exact and explicit a mathematical expression as both logic and sensitivity to the intellectual context within which they were proposed warrant. Specifically, we will look at: 1) the relationship between distance and light-intensity; 2) the relationship between obliquity and light-intensity; and 3) the rule of vertical conservation as both a mathematical device and a regulating principle.

Distance and light-intensity: if the Perspectivist ray-model is taken literally to represent the inverse proportionality between distance and intensity, then each ray will represent an inverse measure of intensity. Suppose, for example, that AB and

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[1] See chap. 4, n. 13.

CB in Figure 15 represent two distinct rays, and suppose that AB is exactly half as long as CB. Further suppose that A and C represent the source-points of light along AB and CB respectively. Then the light-intensity at endpoint B of ray CB will be precisely half the light-intensity at endpoint B of ray AB. However, if we take the ray figuratively to represent *speed* (i.e., distance in a given amount of time), then the original inverse proportionality will be transformed into a direct proportionality. Thus, the two rays AB and CB will each represent a distance traversed in a certain time *t*. In order to traverse CB in that time, the light must go twice as fast as it does to traverse AB in the same time, because CB is twice as long as AB. But since we have assumed a virtual equivalence between speed and intensity, the ray can now be taken as a direct measure of intensity. Accordingly, CB will represent twice the light-intensity of AB.[2]

Obliquity and light-intensity: it has already been established that the Perspectivist view of the relationship between obliquity and intensity can be translated mathematically into a statement of inverse proportionality between the angle of incidence (measured from the normal) and the vertical intensity at the point of incidence. As this angle gets larger, then, the vertical intensity at its vertex will get correspondingly smaller. But, in fact, that is so only if we append the qualification "all other things being equal." These "other things" reduce to the condition that the light must always reach the vertex of the angle of incidence with the same absolute power or intensity, no matter what the angle of incidence. In other words, the light must always have the same speed when it reaches that point, which means that the length of its trajectory (i.e., the ray-length) must be constant.

For example, if the three rays AB, MB, and NB in Figure 15 are all the same length, it follows from the direct proportionality between ray-length and speed that the light traversing all three of them will reach point B with exactly the same absolute speed or intensity. The only thing that can vary is the angle of incidence and, with it, the vertical intensity. As the angle of incidence increases from MBA to NBA, then, the vertical intensity will decrease proportionately. We can translate this inverse proportionality into a direct proportionality simply by taking the complement of the angle of incidence and relating it to the vertical intensity. In that case, as the complementary angle increases from NBO to MBO, the vertical intensity will also increase.

Conversely, as those complementary angles increase, the vertical distance from their endpoints to line BO—measured by the *catheti* MP and NO—will increase correspondingly until the limit is reached along normal AB itself.[3] Moreover, those

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[2] We saw toward the end of Chapter IV that in certain ways the Perspectivist model of light-radiation, taken in terms of virtual motion, was consonant with the Scholastic Aristotelian model of free-fall; in both cases, as we saw, there is an equivalent relationship between the natural efficiency of the trajectory insofar as it is rectilinear, and the acquisition or loss of a body's power or speed along it—power being considered as a direct function of speed. It was commonly assumed by Scholastic commentators that, in free-fall, speed is directly proportional to distance fallen (sometimes in conjunction with the contradictory assumption that speed is directly proportional to time of fall—see Marshall Clagett, *The Science of Mechanics in the Middle Ages* [Madison: University of Wisconsin Press, 1959], 554–56, 678; see also Edward Grant, ed., *A Source Book in Medieval Science* [Cambridge: Harvard University Press, 1974], 280–84). By extension, then, if the ray is taken to represent an equivalent sort of trajectory, the light's intensity or speed along it ought to be directly proportional to the ray's length, so it is quite natural to transform the ray from a representation of distance to a representation of speed.

[3] This of course is in line with the Perspectivist claim that all natural actions, including that of light, are strongest and most efficient along the perpendicular. Hence, the normal represents an absolute limit for the efficiency or power of luminous action.

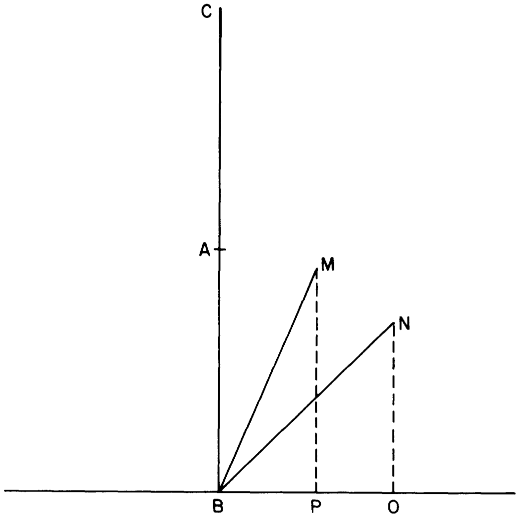

FIGURE 15

catheti will be to one another in the very same ratio as their corresponding angles (i.e., MP:NO = MBO:NBO), provided that the hypotenuse (MB or NB) remains constant. The hypotenuse will certainly remain constant as long as the speed of the incident light remains constant. Therefore, under these conditions, the *catheti* will provide not only a direct linear measure of light-intensity along the vertical, but also a direct way of relating that intensity to obliquity.

The rule of vertical conservation: to understand the crucial significance of this rule as a mathematical device, we need only look at Figure 16. Here, ray AB is taken to represent a direct measure of the speed and therefore the intensity of the light when it reaches point B of impact on surface DF of the denser medium. *Cathetus* AF, meanwhile, represents a direct measure of the vertical intensity or speed of the light upon impact. After impact, the light will have lost some speed, depending on the density of the medium whose surface it traverses. Assume that it loses half. If so, then we know that the upper medium is half as dense as the lower, since we have already supposed that the speed or intensity of the light is inversely proportional to the density of the medium through which it passes.[4] Assume further that, after impact, the light continues along the same trajectory to reach point C′. AB will therefore be twice as long as BC′, so the absolute speed or intensity of the light along BC′ will be half that along AB. Likewise, its vertical speed or intensity at point C′, as measured by *cathetus* C′D, will be half what it was at point B, as measured by *cathetus* AF.

This means that there will have been a loss of power along *both* the horizontal and the vertical. But because a loss of power in the vertical direction is unnatural, it must be offset.[5] That is accomplished by the light's being diverted along BC to point C so that the new *cathetus* CG is equal to AF. Consequently, although there is a loss in absolute intensity (understood in terms of speed, which is a function of

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[4] See chap. 5, n. 2.

[5] See chap. 5, nn. 23–24.

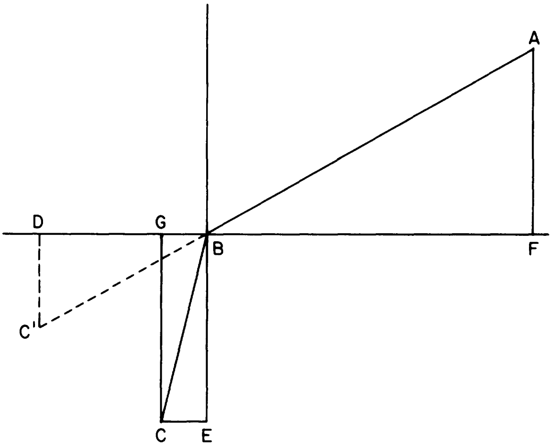

FIGURE 16

the relative lengths of AB and BC), the intensity along the vertical (which is a function of equal lengths AF and CG) is perfectly conserved. The angle of refraction CBE is thus ultimately dependent upon the relative speeds of the light within the two media. Once we know these relative speeds, we can represent them by corresponding ray-lengths, whence we have the corresponding measure of vertical intensity in the *cathetus* of incidence, which is a function of angle ABF and length AB. Then all we need do is measure off length BE on the normal so that it is equal to AF, draw perpendicular EC from point E, and rotate ray BC' toward the normal until it intersects EC. The solution, in short, is more mechanical than mathematical in nature.[6]

The Cosine Relation

The question now is whether, having arrived at a method for determining the angle of refraction in a particular case, we can move from there to a general law. Let EO in Figure 17 represent the interface between two media, the lower of which is the denser, and let MB and NB represent two rays of equal length. According to our previous account, the absolute speeds or intensities of the light along both those rays will be equal, and the vertical intensities will vary as the respective *catheti* MP and NO. Suppose that, after striking the interface at B, the light loses a certain amount of its absolute speed or intensity. As a result, the ray-length after impact (i.e., below EO) will be shorter by that much. The respective angles of incidence will therefore be determined by rotating each of these shorter rays toward the normal until their respective *catheti* CD and AE are exactly equal to the corresponding *catheti* of incidence MP and NO.

In view of the preceding construction, we know that rays MB and NB

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[6] Note how similar in spirit this procedure is to the one outlined by Mydorge on pp. 27–28 above.

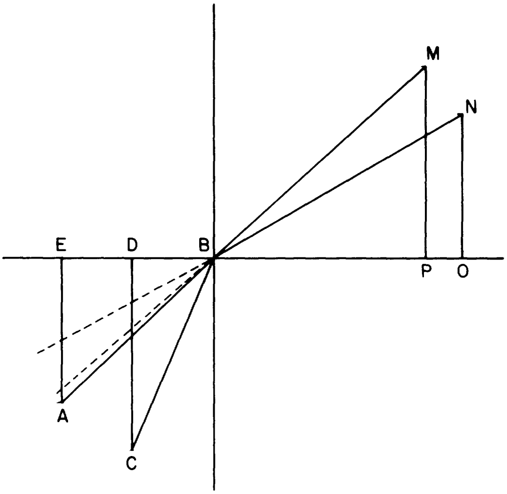

FIGURE 17

are equal and that rays AB and CB are also equal. Therefore, points M and N will both lie on the circumference of a semicircle with radius SB equal to MB and NB. Let the semicircle be represented by SN in Figure 18. Following the same line of reasoning, we can describe through points A and C a semicircle AR with radius BR equal to AB and CB. That lower semicircle will therefore fall short of the larger one by the difference between the

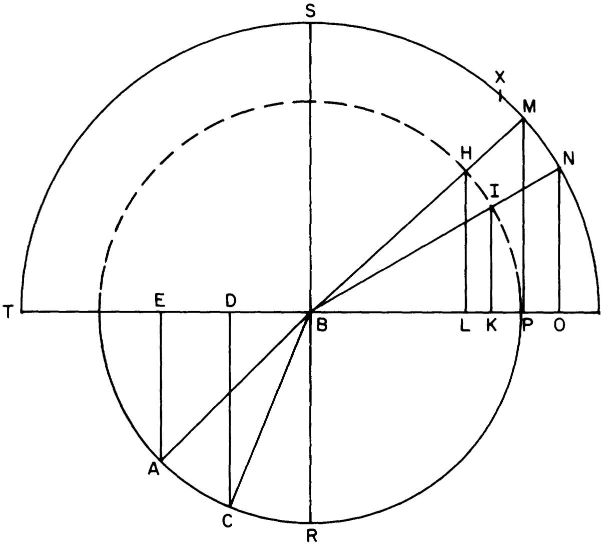

FIGURE 18

respective radii SB (or MB or NB) and BR (or AB or CB), and its continuation along the dotted line within the upper semicircle will fall short by the same amount throughout. This continuation will intersect rays MB and NB at points H and I respectively, so that HB and IB will be equal to AB and CB. Accordingly, HM and IN, which are equal to one another, will represent the difference between radii SB and BR.

We know from the rule of vertical conservation that MP and NO must be equal to DC and EA respectively. We also know from Euclidean geometry that HL and IK are proportional to MP and NO. In short, MP:HL = NO:IK. But if that is so, it must follow that CD:HL = AE:IK, because CD is equal to MP, and AE is equal to NO. This is tantamount to a cosine-law, for within full circle ARH, CD is proportional to the cosine of angle CBR, HL is proportional to the cosine of angle HBS, AE is proportional to the cosine of angle ABR, and IK is proportional to the cosine of angle IBS. Therefore, we seem to have established as a general law that the cosine of the angle of incidence is constantly proportional to the cosine of the angle of refraction—i.e., cosine i:cosine r = constant.

From Cosines to Sines

As satisfactory as this cosine-law may seem at first glance, it does not hold up under closer scrutiny. Perhaps the most obvious problem is that it is inconsistent with the rule of vertical conservation along the normal itself. SB and BR are obviously not equal, even though they should be.[7] That particular inequality opens up yet a more serious problem. As we continue to decrease the obliquity of incidence by rotating ray MB toward S, we will eventually reach a point X at which the *cathetus* dropped from X to EO will be equal to BR. But in such a case, the light traveling along ray XB would necessarily refract along BR if the rule of vertical conservation were to apply. Worse yet, when we continue to decrease the obliquity beyond point X toward S, the *catheti* will actually *exceed* BR, so that even if the light were to refract along BR, the rule of vertical conservation would be flouted. So the cosine-law is not general after all, because, no matter how small the difference in length between incident and refracted rays, there will always be some dead space SX within which incident light will always refract along the normal BR. But that contravenes both the rule of vertical conservation and experience.

The solution to this problem is simplicity itself. All we need do is rotate the circle 90 degrees clockwise (as represented in Figure 19), so that RS becomes the interface between the two media, and EO becomes the normal. Thus, AB will represent an incident ray that is refracted toward normal EO

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[7] That is, if the rule of vertical conservation were to be truly universal, the intensity/speed of the light along the normal ought to be perfectly conserved throughout. But of course to accept such perfect conservation along the normal would be incompatible with accepting (as the Perspectivists did) that some loss *always* occurs when light passes into a denser medium, no matter what the angle of the incident trajectory.

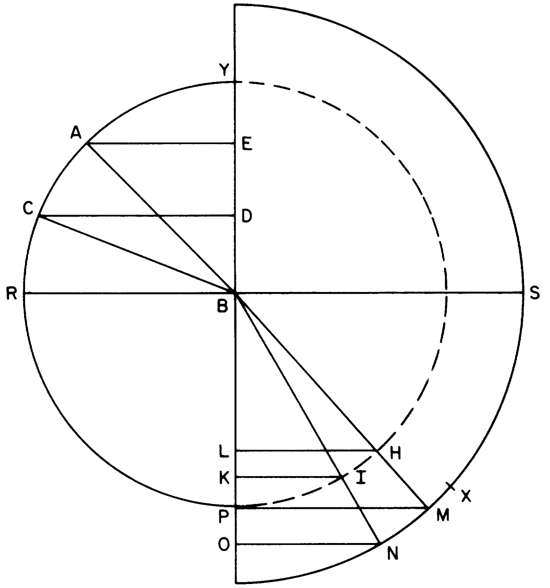

FIGURE 19

along BN, whereas CB will be refracted toward normal EO along BM. From this it follows that the denser medium must lie below RS. But since the incident ray is now *shorter* than the refracted ray by HM or IN (both equal to one another), the light must have gained speed in passing into that denser medium. The relation between speed and density has therefore been inverted. Meantime, the rule of conservation has been converted. It is now along the *horizontal* instead of the vertical. Everything else that was true for the previous case will extend to this one. AE and CD will still be proportional to IK and LH, except that now they represent sines instead of cosines. For that reason, the previous cosine formulation must be revised to make the sine of the angle of incidence constantly proportional to the sine of the angle of refraction. In other words, the law must take the form sine i:sine r = constant.

What makes this change "right" in the long run is that it solves the problem of dead spot SX, because the light would have to travel along RS or below it in order to refract along BX or any other line between it and BS. But we know full well that light traveling along surface RS could not penetrate it and therefore could not refract. Furthermore, there is no dead spot between A and Y in Figure 19. No matter how close to normal YB the incident ray may verge, its refracted counterpart will never reach extension BO, and this agrees with both intuition and experience. Thus, by the simple expedient of inverting the relation between speed and density and translating the direction of conservation from the vertical to the horizontal, we have succeeded in making the law truly general according to the constant proportionality of sines.

Descartes Revisited

In case the real point of the preceding analysis has been lost in the crush of lines, angles, and proportional relationships, let me state it as simply and forcefully as I can. Briefly, we have just made an easy, natural, and logical progression from a small set of first principles, through a cosine relation, to the sine-law. That we did so is a testament more to common sense than to genius or deep reasoning. All it really required was the ability to grasp certain basic tenets of Perspectivist optics, draw out their manifest implications, give them explicit mathematical expression, and then follow them to their logical conclusion. Surely, the very naturalness and ease with which we made this progression is warrant enough to suppose that Descartes could have made it as well. It is, of course, a major leap from demonstrating that he *could* have to concluding that he actually *did*. But the likelihood that he did seems overwhelming when we compare the form of his derivation and proof with that of the derivation and proof we have just constructed. A brief glance at Figures 8 and 19 will suffice to show that they are virtually identical.

That in itself is not conclusive evidence. But the case gains a good deal of strength in view of the fact that many, if not all, of the fundamental logical problems in his account—which seemed to indicate the *ad hoc* nature of his procedure—are not only explained, but actually *necessitated*, by his having followed the line of reasoning outlined above. First, the rule of horizontal conservation makes perfect sense in light of that reasoning, because, without just that rule in that very form, the consequent proof (in the form of the cosine-law) lacks true generality (supposition 3, p. 31 above). So the rule itself is anything but *ad hoc;* its supposed justification in the "law" of inertia only makes it seem so.[8] Once that rule is accepted, however, it is necessary to assume that the light travels more swiftly through the denser medium. Otherwise, it would not refract toward the normal, and experience shows that it does. So the assumption that light actually gains speed in the denser medium makes perfect sense within its proper theoretical context (supposition 4, p. 31 above).

All of this is in turn contingent on the assumption that light actually moves in space and time, which seems to contravene Descartes's basic contention that light is transmitted instantaneously. But we have seen an obvious precedent for this apparent contradiction in the Perspectivist account. Just as did Descartes, the Perspectivists based their account on an analogy between light-transmission and projectile motion. And just like him, they justified the use of that analogy on the grounds of an implicit but clear distinction between virtual and real explanations.[9] Accordingly, both Descartes and the Perspectivists accepted that light acts *as if* it were matter in motion without conceding that it actually *is* matter in motion (supposition 1, p. 31 above).

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[8] See chap. 3, n. 32. There is in fact no convincing reason to assume that Descartes's rule of horizontal conservation, and thus his proof of the sine-law, were actually adduced from his "law" of inertia in *Le Monde.* If not, then the "law" itself only serves as an *a posteriori* justification, not an *a priori* basis, for the rule of horizontal conservation.

The similarities between the Perspectivists and Descartes extend even to the specific mechanical analogies they used. Both, of course, used the standard analogy of a projectile passing into and through a fluid. But since that analogy was canonical from at least Ptolemy's time, the fact that both shared it is hardly telling. Far more telling is the striking similarity between Alhazen's ball-and-plank example and Descartes's tennis-ball-and-gauze-net example, because that particular analogy seems to have been original to Alhazen. The very use of this specific kind of analogy by both is also striking, for it allowed both to treat refraction as a surface phenomenon and thereby to reduce it dynamically and kinetically to an instantaneous act (supposition 2, p. 31 above). That way they were both able to avoid a fundamental problem posed by the standard fluid analogy: that the projectile in such a case should refract toward the normal in a curve because of a gradual, rather than a sudden and complete, loss of momentum.[10]

Finally, as we saw earlier, both the Perspectivists and Descartes used much the same sort of vectorial technique for analyzing the kinetic and dynamic properties of light in refraction. This is significant in two crucial ways. First, the technique itself harks back directly to Alhazen's *De aspectibus,* whence it entered into the western Latin tradition to be taken up by Witelo and Kepler. Second, it points the analysis itself inexorably toward the use of a conservation rule. All of this may be coincidence, to be sure. There is no way of proving otherwise, short of a point-by-point avowal by Descartes that he actually followed the course we have reconstructed. This we do not have. But what we do have in the form of circumstantial evidence, supplemented by judicious inference, gives considerable force to the conclusion that he did.

Having reached this conclusion, we are now in a position to clear up some long-standing debates about Descartes's optical research. First, although his analysis of refraction is indeed incoherent in an *absolute* sense, it is nonetheless perfectly logical in a relative sense—that is, granted its suppositional basis. Thus, contrary to most scholarly opinion, Descartes's rationale in the *Diotrique* is neither *ad hoc* nor arbitrary. In fact, much of the appearance of arbitrariness in that rationale stems from the analogies Descartes used to justify certain of his logical steps.[11] Second, Descartes's line of reasoning in the *Dioptrique,* far from meretricious, actually does provide a clear and inexorable path to the sine-law. This gives infinitely greater weight to the claim that Descartes discovered the law independently than to the counter-claim that he plagiarized it. Finally, if Descartes's analysis of refraction serves as an example of his scientific method, then it is obvious that this method was not completely new or revolutionary. In a certain sense it was a permutation—a streamlined version, if you will—of the Scholastic method it was intended to supersede. The lesson to be learned from this is as simple as it should be self-evident. Descartes did not necessarily practice what he preached. In attempting to understand his scientific achievements, therefore, we would do far better to concentrate upon his actual scientific practice than upon his philosophical ruminations about it.

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[9] The Scholastics exploited this distinction in a wide variety of ways when offering analogical "explanations." For an illustrative account that is slanted specifically toward the Perspectivists, see my "Getting the Big Picture."

[10] In avoiding this particular problem, both Descartes and the Perspectivists also avoided any implication that light consists of bodies actually moving in space and time through various material media.

[11] See Eastwood, "Descartes on Refraction," 500–502.

VII. THE CARTESIAN LEGACY RECONSIDERED

To deny Descartes total originality in framing his theory of light and refraction is by no means to slight his creative genius. It is merely to put that genius into true proportion as an uncommon knack for clarifying and simplifying existing ideas. Certainly it was this knack more than anything else that helped Descartes accomplish what he did in optics by enabling him to find and exploit certain logical possibilities raised but never recognized by the Perspectivists. In one sense, then, Descartes's theory of light and refraction was a culminating step within the Perspectivist tradition. In another sense, though, it was more than that. It was the beginning of a new tradition as well, for, like the Perspectivists, Descartes also raised but failed to recognize certain logical possibilities in his own work, leaving it to others after him to find and exploit them. Without assessing his optical contribution from both standpoints, as a culmination *and* a beginning, we can never fully understand or appreciate its importance as a springboard for later developments in both optics and epistemology.

Cartesian Light-Theory as a Culmination

From a modern viewpoint, of course, the only satisfactory thing about Descartes's proof of the sine-law is its conclusion; the rest can be summarily dismissed as wrong. From the principle of horizontal conservation to the assumption that light finds easier passage when it enters a more optically dense medium, the basis of Descartes's reasoning is wholly unacceptable to us now. Why it was not so to him then is because, like his Scholastic predecessors, he was unable to conceive of light in terms of actual motion through space. That inability stemmed from an even deeper inability to conceive of space itself. Extension was synonymous with matter for Descartes, and space, being immaterial, could have no extension. Since it must always have an extent, motion could therefore not conceivably occur in space.[1] So, as far as Descartes was concerned, motion was a function of relative material displacement rather than of free movement through a void that could not possibly exist anyway.[2]

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[1] For Descartes's equation of matter and extension and his consequent denial of the possibility of a void, see *Meditations* 3 (*AT,* 7: 42–45 [Latin version] or *AT,* vol. 9, pt. 1, pp. 34–34 [French version]); *Le Monde* 5 (*AT,* 11: 33–36); and *Principes de la philosophie* 1, 53 and 2, 10 (*AT,* vol. 9, pt. 2, p. 48 and 9.2: 68).

[2] Descartes defines motion as "LE TRANSPORT D'VNE PARTIE DE LA MATIERE, OU D'VN CORPS, DU VOISINAGE DE CEUX QUI LE TOUCHENT IMMEDIATEMENT" (*Principes* 2.25 [*AT,* 9.2: 76]).

If Descartes was loath to admit that light involved the passage of matter through space, he was no less loath to admit that it was a function of relative displacement. Why not is easy enough to comprehend in view of the problems such an assumption raises. In the first place, it is difficult to believe that light could move as quickly as it seems to do if its matter were constantly interfered with by the matter it would have to displace in the course of radiation.[3] Equally difficult to believe is that light could travel as far as it does under such conditions, but that under those same conditions it could continue to move in perfectly straight lines for any appreciable distance is impossible to believe.[4] Thus, unable to accept absolute space or relative displacement as grounds for its motion, Descartes had no choice but to deny that light could travel in any way whatever. This forced him to accept the instantaneity of its transmission and, having made that choice, to make the best of it.

There is no need to belabor the point that, by restricting himself to the same basic theoretical framework as the Perspectivists, Descartes could do no more than build upon the foundations they had already laid.[5] In short, he could not substantially change the grounds of analysis without changing that framework. Given those limitations, Descartes accomplished as much as could fairly be expected of him. He distilled the Perspectivist theory to its essential elements, streamlined it to almost perfect simplicity, made it rigorously explicit in both mathematical and physical terms, and from there carried its logical and mathematical implications as far as they would go—straight to the sine-law. That was not quite far enough, though, because the line of reasoning he necessarily followed just as necessarily entailed logical inconsistencies. At worst, then, Descartes's attempt to achieve a coherent account of light was a dismal failure. At best, however, it was a limited success, as even Newton grudgingly admitted.[6]

Had it simply been deemed a failure, Descartes's account of refraction would surely not have exerted the powerful influence it did. What gave it its cachet was its somewhat paradoxical status as both a complete failure and a partial success at the same time. As a failure, on the one hand, it revealed the fundamental inadequacy of the Perspectivist approach to optics. No amount of change within the basic theoretical framework of that approach was likely to improve the situation, so the framework itself had to be changed.[7] In other words, Descartes's inability to achieve a satisfactory explanation of light was a clear signal to many in his day that the science of optics needed an entirely new theoretical footing. As a partial success, on the other hand, his account not only held out the promise of such an explanation but also pointed the way, through its strong mechanistic overtones, to the new theoretical footing upon which that promise might be fulfilled.

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[3] If light were to move by displacement, then the light-particles would have to push through the Aereal medium, which would resist accordingly. That would either slow the passage to nothing or divert it markedly in very short order. However, those of Descartes's contemporaries who believed in a finite velocity for light believed that its passage was so swift as to be imperceptible (see chap. 4, n. 27).

[4] According to Descartes's theory of vortices, although matter strives to move in straight lines, it is unable to realize such rectilinear movement on account of the reaction of surrounding matter (see chap. 3, n. 32).

[5] That is, by conceiving of the universe as a real plenum, Descartes was as much an Aristotelian as the Perspectivists.

[6] In the 1728 edition of his *Optical Lectures,* Newton confessed with typical English understatement that Descartes had demonstrated the sine-law "not inelegantly"—a monumental tribute from one who consistently opposed Descartes's physical principles (quoted from Sabra, *Theories of Light,* 300).

Where Descartes seemed to have been most successful was in treating light as if it were matter in motion, even though his philosophical scruples kept him from assuming that it really was.[8] To save those scruples, he had relied heavily on the Scholastic distinction between virtual and real explanations, all the while narrowing the gap between the two. This point is crucial, because, in failing to bridge that gap, Descartes had left a variety of loose logical ends in his account. In narrowing the gap as much as he did, however, he made its crossing almost inevitable. All that was left to do was to make the short and obvious leap to making literal the virtually kinetic model of light he had so carefully constructed. Therefore, one of Descartes's most significant contributions to seventeenth-century optical theory lay in the all-important challenge he posed for his immediate successors: to effectively transform light into matter in real motion. This challenge played a crucial guiding role in the development of light-theory for a good forty years after the publication of the *Dioptrique.*

Toward a Kinetic Theory of Light

Descartes's challenge was met in two fundamental ways. The first of these ways—in logical, if not chronological, order—was founded upon the Cartesian supposition that the world is a plenum. The problem in this case was to preserve the transparent medium's essential continuity while at the same time allowing its constituent particles to undergo some sort of spatial translation. Although Robert Hooke and Ignace Pardies grappled in their own ways with this difficulty, it was Christiaan Huygens who reduced it to its clearest terms.[9] "One cannot doubt," he remarked at the outset of his *Traité de la lumière,* "that light consists in the movement of some kind of matter."[10] But, he cautioned soon afterward, this movement "cannot be by the transport of any matter which comes to us from [a luminous source] in the way that a ball or an arrow traverses the air."[11] The problem for Huygens was thus to reconcile these apparently contradictory positions, and his solution was as simple as it was ingenious. He merely transformed the perfectly inelastic particles of Cartesian Air into almost perfectly elastic particles of Ether.[12]

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[7] Such a change was already afoot insofar as many thinkers of the time were flirting with atomism (see my "Galileo's Theory of Indivisibles: Revolution or Compromise?" *Journal of the History of Ideas* 37 [1976]: 578–85).

[8] The crux of Descartes's theory of refraction was the idea of easier or more difficult traversal, which translates most obviously into a difference in speed of transmission. That Descartes nonetheless avoided such a translation was due to his conscious effort not to put light into actual motion, an effort dictated by his deepest philosophical convictions.

[9] Hooke published a modified Cartesian account of light and color in his *Micrographia* (1665; reprint ed., New York: Dover, 1961), 47–64. This account was based on the idea that light is transmitted in pulses through the transparent medium. At the same time, Father Pardies was working out his own pulse-theory, which he left unpublished. Huygens acknowledged both Hooke and Pardies in *TL* 1, 18. For a fine, detailed account of the historical background to both Hooke's and Pardies's theories, with especial focus on the contribution

Minor though it seems, this change led to a major revision of Descartes's theory, and it did so with a minimum of damage to the theory's fundamental structure. For instance, like Descartes, Huygens supposed light to be caused by rapid and successive centrifugal nudges at the surface of luminous sources.[13] He also supposed with Descartes that these sources are surrounded by an envelope of perfectly contiguous particles constituting the transparent medium.[14] However, since he had now endowed those particles with elasticity, the transmission of luminous pressure through them was bound to take a form different from that postulated by Descartes. In fact, as Huygens demonstrated, it had to take the form of longitudinal oscillations or waves passing over time through the Ether.

The gist of Huygens's argument is this: suppose that A in Figure 20 is one particle of Fire on the surface of the luminous source, and that BCDEF represents a rectilinear succession of Ether particles, as pictured in 20a. When A pushes outward against that succession, particle B will immediately give in, allowing itself to be squeezed together, as pictured in 20b. Being elastic, however, it will then recoil, communicating the initial pressure to particle C, which will undergo the same contraction-recoil sequence, and so on down the line (20c and d). Thus, the initial push exerted by A against the line of particles will be transmitted in the form of successive oscillations, particle by particle along that line.[15] Such an oscillatory transmission will

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of Hobbes, see Alan E. Shapiro, "Kinematic Optics: A Study of the Wave Theory of Light in the Seventeenth Century," *Archive for History of Exact Sciences* 11 (1973): 134–218; see also Sabra, *Theories of Light,* 185–97. For Huygens, see Shapiro, 218–44 and Sabra, 198–230.

[10] "L'on ne sçauroit douter que la lumiere ne consiste dans le mouvement de certaine matiere" (*TL* 1, 2 [ll. 31–32]). Huygens adverts somewhat later to Olaus Roemer's famous experimental determination of the speed of light in 1679 at the Paris Observatory (see *TL* 1, 7–9). According to Sabra, Huygens had already accepted temporal transmission of light before Roemer's determination (*Theories of Light,* 202–203).

[11] "On comprend bien que quand nous voyons un objet lumineux, ce ne sçauroit estre par le transport d'une matiere, qui depuis cet objet s'en vient jusqu'à nous, ainsi qu'une bale ou une fleche traverse l'air" (*TL* 1, 3 [ll. 22–25]).

[12] "Rien n'empêche que nous n'éstimions les particules de l'ether estre d'une matiere si approchante de la dureté parfaite et d'un ressort si prompt que nous voulons" (*TL* 1, 12 [l. 32]–13 [l. 2]). Note that Huygens ties the elasticity of the Ether particles to their hardness, so that the harder they are, the more powerfully and quickly they rebound.

[13] "Car soit qu'on regarde sa [i.e., light's] production, on trouve qu'icy sur la Terre c'est principalement le feu et la flamme qui l'engendrent, lesquels contienent san doute des corps qui sont dans un mouvement rapide" (*TL,* 1, 2 [l. 32]–3 [l. 3]; see also p. 15).

[14] "Et cette matiere etherée, . . . consiste en des particules qui se touchent prez à prez" (*TL,* 3, 28 [ll. 18–19]).

[15] See *TL* 1, 11–12.

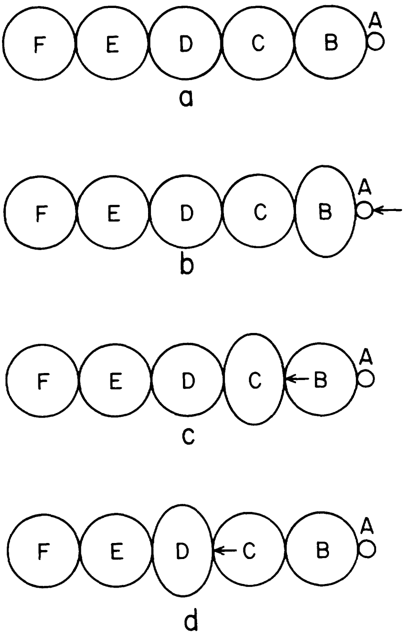

FIGURE 20

necessarily take time, since it obviously takes some time for each particle to undergo contraction and recoil.[16]

Extending this analysis to the entire envelope of Ether, Huygens was finally drawn to conclude that the gross result of such particular cases of oscillation would be a succession of spherical wave-fronts emanating from luminous points and consisting of alternate condensations and rarefactions in the Ether.[17] These spheres of propagation could in turn be analytically resolved into rays along which individual oscillations pass.[18] Accordingly, light would be capable of traveling radially over great distances in as short a time as you please, depending on the elasticity of the Ether particles. The shorter their cycle of contraction and recoil, the swifter the wave-front's overall passage.[19] Meanwhile, the individual particles that support this passage would have real but highly restricted movement, insofar as they might undulate back and forth to a certain extent during their cycle of contraction and recoil. Still, their average locations would never change, nor would the particles themselves ever move independently through space, since they had to remain perfectly contiguous throughout.[20] The motion of matter in Huygens's system was thus at bottom a function of relative displacement. However, having shown that the wave-fronts would necessarily move with a finite velocity, Huygens was free to treat those fronts and their component rays in actual kinetic and mechanistic terms. On that basis, he was driven to conclude against Descartes that the motion of light was impeded rather than eased in denser media.[21]

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[16] "Ce qui fait voir que ces matieres obeïssent à leur rencontre, et se restituent; à quoy il faut qu'elles emploient du temps" (*TL* 1, 12 [ll. 27–29]).

[17] See *TL* 1, 15–18 for Huygens's explanation of the generation of these wave-fronts out of individual lines of transmission. One of the most significant features of Huygens's analysis of these wave-fronts is his composition of primary fronts out of secondary ones in order to reinforce the overall power of the front as it is transmitted over great distances (see *TL* 1, 17–18).

[18] See *TL* 1, 19 (ll. 8–17). Just as for Descartes, so for Huygens, "rays" represented lines of mean distribution through non-rectilinearly arrayed particles (*TL* 1, 14 [ll. 2–7]).

[19] This of course follows from his having related speed of transmission to the hardness and elasticity of the individual particles of Ether and arguing thence that the harder and more elastic they are, the more quickly they rebound after compression.

The second way in which Descartes's challenge was met was to throw philosophical caution to the winds and assume that light actually consists of particles hurtling through space. A total and overt repudiation of Cartesian principles, this way was taken initially by Fermat and later by Newton.[22] As we have seen, Fermat's objection to Descartes's proof came to ground in his opinion that the proof was equivocal in certain critical respects. To begin with, Fermat distrusted the principle of horizontal conservation of motion. Moreover, he doubted that an equivalence could be safely drawn between inclination to motion and motion itself. Finally, he could not believe that light somehow finds easier passage through denser media.[23] To resolve these doubts at once, Fermat chose to treat the action of light as if it were identical in all essential points to that of a projectile.[24] In short, he decided to carry the mechanistic implications of Descartes's analysis to the literal extreme. The result was his proof of the sine-law from the Principle of Least Time.

The grounds of Fermat's proof can be summarized as follows: let diameter AB of circle ACBG in Figure 21 represent the interface between two media, the one lying above it being the rarer.[25] Assume that the speed of light is inversely proportional to the density of the medium through which it passes so that, for example, if the upper, rarer medium were half as dense as the lower one, light would move twice as fast through it.[26] Assume further

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[20] When the Ether particle is pushed and thereby compressed, its center must move a bit in the direction of the push. This movement will occur particle by particle throughout the succession. In restoration, moreover, if the push is balanced by a counter-push, the centers will also move in that direction—hence, a perfectly symmetrical oscillation of that center-point.

[21] See *TL* 3, 30. Huygens ties optical density to the amount of matter contained by the transparent body, so that the more matter there is, the more the wave-front is interfered with.

[22] It is simplistic to characterize Newton's theory of light as a straightforward particle-theory, because, as we shall see in a moment, it partook of some of the characteristics of a wave-theory.

[23] See chap. 3, nn. 46–48.

[24] Note, however, that for Fermat light did not act like a projectile in one critical respect: its change of course at the interface was still taken to be instantaneous rather than gradual. Newton, however, did make that change gradual by assuming that, when striking the surface of refraction, the light actually moves through a thin gradient of Ether whose density decreases the closer it is to the refracting surface. Thus, Newton insisted that the bending of light in refraction takes some tiny amount of time (see *Opticks* [New York: Dover, 1952], bk. 3, Query 20, p. 350).

[25] See *OF*, 2: 457–63.

[26] Fermat makes this assumption explicit in *OF*, 2: 459.

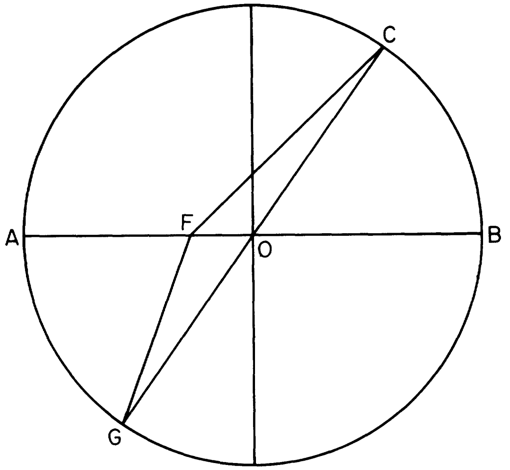

FIGURE 21

that, in moving from point C to point G through interface AB, the light will act as efficiently as possible. If its efficiency were a function of the Least-Lines Principle, the light would necessarily follow straight line COG through the center of the circle, since that is the shortest trajectory possible. We know that it does not follow this trajectory, so obviously the gauge of efficiency is not spatial brevity.[27] What if we assume instead that the gauge is temporal? What sort of path would light take if its passage through the media required the least possible time? Fermat's search for a definitive answer took several years, but by the early 1660s he was finally able to demonstrate that the most temporally efficient path is in fact determined by the sine-law. In other words, if the ray is taken to represent a path in *time* rather than in space, the sum of times CF and FG in Figure 21 will be the smallest possible when path CFG yields a constant sine-relationship for the angles of incidence and refraction.

The basic tenor of Fermat's demonstration is quite easily grasped. Suppose that the light moves twice as fast through the upper medium in Figure 21 as it does through the lower one. Then it will travel from C to O in half the time it takes it to travel from O to G. Let its time of travel over CO be t. Therefore its time of travel over OG will be $2t$, and its overall time of travel will be $3t$. Meanwhile, in traveling from C to F, it will take longer than t, since distance CF is greater than distance CO. Let the excess amount of time be m. Meanwhile, in passing from F to G it will take less time than it would to pass from O to G. Let the difference be n. The problem then was to demonstrate that the combination of $t + m$ and $2t - n$ dictated by the sine-law will yield the smallest possible sum—i.e., $3t + m - n =$ minimum, when the sine-law holds.[28]

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[27] That Hero of Alexandria's Principle of Least Lines (see Appendix D) does not apply to refraction is self-evident. However, despite its failure to cover all cases, that principle was crucial as a paradigm inasmuch as it pointed Fermat's analysis toward the use of a related extremal principle.

With Fermat and Huygens we have come a long way from the Perspectivists—to the very threshold of modern optical theory in fact. The next logical and chronological step would be taken by Newton, who proposed a hybrid particle/wave-theory in which light is composed of corpuscles shooting with incredible velocity though an Ethereal fluid composed of miniscule particles.[29] Interacting with this fluid in ways that depend upon the forces of its constituent particles, light-corpuscles can be diverted from their normally rectilinear trajectory to reflect, refract, or diffract.[30] Since they interact with the Ether, moreover, those corpuscles necessarily affect it, one of the most intriguing and significant results of which is the production of transverse waves.[31] This is of supreme importance, because, as it turned out, the transverse wave was the wave of the future for optics.[32]

In Newtonian retrospective, then, the tandem development of Fermat's and Huygens's theories of light was nothing short of revolutionary, because it marked a definitive turning-point in the history of optics. The distinction between real and virtual explanations in the physics of light had finally been abolished, which in turn made it possible to lift all the vexing theoretical constraints imposed by the Perspectivists. As a result, the grounds of optical analysis had been totally shifted from the static formalism of medieval theory to the kinetic materialism of modern theory. The ray had thus been transformed from a purely spatial to a purely temporal abstraction, and the science of optics had been given a truly systematic basis in the Least-Time Principle.

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[28] For a discussion of Fermat's actual method of proof, see Sabra, *Theories of Light,* 144–52; see also Kirsti Andersen, "The Mathematical Technique in Fermat's Deduction of the Law of Refraction," *Historia Mathematica* 10 (1983): 48–62. Huygens accepted Fermat's proof and offered a précis of it in *TL* 3, 39–41.

[29] See *Opticks* 3, Queries 21 and 29, pp. 350–52 and 370–71.

[30] Aside from the normal phenomena of reflection and simple refraction, several new optical phenomena had been observed by the time of Newton. Diffraction at the edges of opaque bodies, which Newton called "inflexion," was first described by Francesco Grimaldi in his *Physico-mathesis* of 1665. The phenomenon of diffraction through thin plates, which has taken the name of "Newton's rings," was observed by Hooke, who attempted an explanation of it on the basis of a theory of pulses. Finally, the fact of double refraction in Iceland Spar was first described by Erasmus Bartholinus in 1669. Newton devoted most of his efforts in the *Opticks* to integrating these phenomena into the general framework of optics on the basis of his theory of light-particles, Ether gradients, and particle to particle forces. Out of that effort came one of his most crucial contributions to the later development of optics: the idea of polarity. See, e.g., *Optics* 3, Queries 1–5, 25, 29, and 31, pp. 339, 354–61, and 371–74. Newton's explanation of the dynamics of refraction required that, as Descartes claimed, light gain speed when it enters an optically denser medium (see *Optics* 2, pt. 3, proposition 10, pp. 270–76; see also Sabra, *Theories of Light,* 299–312). This claim was finally disproved in the middle of the nineteenth century.

[31] To explain the rings of color in thin glass plates, Newton suggests that "nothing more is requisite for putting the Rays of Light into Fits of easy Reflexion and easy Transmission, than that they be small Bodies which by their attractive Powers, or some other Force, stir up Vibrations in what they act upon, which Vibrations being swifter than the Rays, overtake them successively, and agitate them so as by turns to increase and decrease their Velocities, and thereby put them into those Fits" (*Opticks* 3, Query 29, pp. 372–73). These "Vibrations" are of course just like water-waves.

[32] The theory that light consists of transverse waves of varying lengths was introduced by Thomas Young in the very early nineteenth century and became standard by the middle of that century, largely through the efforts of Augustin Fresnel.

Still, while there is no denying their magnitude and significance, these changes can hardly be called revolutionary when viewed in proper historical perspective. Every one of them was clearly portended in Descartes's analysis of light and refraction, which was itself implicit in the Perspectivist account. The Perspectivists, after all, had developed a quasi-kinetic account of light that foreshadowed both the particle-theory and the longitudinal wave-theory.[33] The Perspectivists had gone a considerable way, as well, toward transforming the ray from a spatial to a temporal entity; and by attempting to explain refraction in terms of easiest passage, they had laid the necessary groundwork for the Least-Time Principle.[34] In a way, then, it is fair to say that Huygens, Fermat, and even Newton only brought to maturity seeds sown by the Perspectivists and nurtured by Descartes. And that is precisely what makes the Cartesian theory so important in the history of optics. It forms the crucial middle term in a logical progression from medieval to early modern theory. Without that middle term, the progression would have been, if not impossible, at least unlikely.

The Epistemological Consequences

So far we have seen that the development of early modern optical theory was rooted in the seemingly straightforward transition from a virtual to a real kinetic model of light. We have also seen that this transition and its attendant benefits were so clearly indicated in the Perspectivist account of refraction as to be almost self-evident. Yet the transition took a full three-and-a-half centuries to occur. Why was it not made sooner if it was so logically palpable and straightforward? One reason has already been suggested. To suppose that light actually consists of matter in motion entails an atomistic conception of the universe. In short, it demands the acceptance of real space, at which even Descartes bridled.[35] More than ontological principles were at stake, though. Epistemological principles were too. In fact, the transition to a fully kinetic theory of light had such overwhelming epistemological consequences that, in the face of them, the wonder is not that the transition took so long to take place, but that it ever occurred at all. To appreciate the import of this point, we must turn to a brief examination of the Perspectivist theory of vision.

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[33] That the quantization of light by the Perspectivists suggests a particle-theory needs no explanation. Of the longitudinal wave-theory we have a clear suggestion in Bacon's theory of species-multiplication in successive spheres. Moreover, the Perspectivists as a whole divided light-radiation into primary and secondary or accidental radiation, such that each point on the primary ray generated accidental rays in a spherical pattern (see, e.g., *PC* 1.14, 89; *WP* 2, definition 4, p. 61; and *DMS* 2.2, 102 [l. 116]–104 [l. 141]). This notion may well have been picked up by Huygens, modified, and applied to his theory of primary and secondary wave-fronts. Finally, Bacon expressly draws an analogy between sound and light, sound being thought to propagate according to longitudinal fronts through the air (*DMS* 4.3, 225 [ll. 58–71]). This analogy is also used by Huygens in *TL* 1, 11.

[34] See n. 27, this chapter.

[35] Huygens, following Descartes, attempted to avoid postulating space and atoms by reducing Ether particles to smaller constituent particles, which themselves could be further subdivided *ad libitum* in order to explain their elasticity (*TL* 1, 12–13). By so doing, however, he was merely deferring the inevitable leap to true atomism.

Perspectivist optical theory was designed with one basic end in mind: to explain how we get internal visual impressions of external physical objects.[36] Accordingly, the Perspectivists conceived of vision as a process in which the visible characteristics of such objects are conveyed to the eye and thence into and through the brain. Three things were deemed necessary for this process to occur. First, the objects must possess color, because color was thought to be the inherent ground of any object's visibility.[37] Second, in order to become *actually* visible, colors have to be illuminated, so the primary visual function of light is to actualize the inherent visibility of physical objects.[38] Third, a suitable pathway is necessary for the conveyance of illuminated color to the eye and thence into the brain. For example, the medium between the object and the eye needs to be transparent.

At first glance, these conditions might seem insufficient. For a start, we see considerably more than color when we see a physical object. We also see size, shape, smoothness, and so forth. This the Perspectivists openly admitted, adducing altogether twenty-two such visible attributes.[39] However, the Perspectivists were astute enough to realize that most of those kinds of physical attributes could not actually get into the eye or brain unless they were brought there by the object itself. The rationale is simple enough. Having immediate visual apprehension of an object's size means having the size itself in the eye, which in turn means having either the object itself or an exact replica in the eye.[40] To avoid this absurdity, the

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[36] For a detailed account of Perspectivist visual theory see my "Getting the Big Picture." See also A. I. Sabra, "Sensation and Inference in Alhazen's Theory of Visual Perception," in *Studies in Perception,* ed. Peter Machamer and Robert Turnbull (Columbus: Ohio State University Press, 1978), 160–85.

[37] This idea harks back to Aristotle's dictum that color is the "proper object" of sight and that all other visual perceptions are by-products, among which Aristotle includes the so-called common sensibles: movement, rest, number, figure, and magnitude (see *De anima* 2.6).

[38] The Perspectivists parted ways with Aristotle over the function of light. For him, light actualized the potential transparency of certain media, thus making them capable of color-transformation (*De anima* 2.7.418b4–13). For the Perspectivists, however, light was not only per se visible, but it mixed with the color of objects to give that color power to radiate and thereby to be seen (see *DA* 1.39, 22–23, and *WP* 3.1, 85).

[39] These physical attributes were called "visible intentions" by the Perspectivists. Aside from light and color, they comprise distance, place, corporeity, shape, size, continuousness, discreteness or separation, number, motion, rest, roughness, smoothness, translucency, density, shadow, indistinctness, beauty, ugliness, and similarity or difference. See *DA* 2.2.15, 34; *WP* 3, supposition 2, p. 84; and *PC* 1.55, 134–36.

[40] The Greek atomists in fact explained vision in terms of the reception of exact replicas or "eidola," which were atom-thick skins shucked off from the surface of objects and projected to the viewer (see Lindberg, *Theories of Vision,* 2–3). Aristotle rejected this theory for a number of reasons, one of which was the fact that direct contact between an object and the eye does

Perspectivists assumed quite reasonably that those attributes exist only *virtually* in the eye when seen. However, even granted this point, the Perspectivists still had to face the question of how such virtual attributes could reach the eye. In response, they claimed that the only vehicles that can possibly convey such attributes are light and color. After all, in taking on the true light or color of external objects, the eye suffers no essential change whatever, whereas if it were to assume the true shape or size of those objects it would be very much altered indeed.

In Perspectivist theory, then, light and color are the primordial data of vision. As such, they share the same essential properties. Both, for example, are qualitative and therefore purely formal. Both are inherent to their physical subjects, light rendering them luminous, color making them red, green, etc. Both can replicate themselves spherically through transparent media, *lux* by means of *lumen,* illuminated color by means of a similar kind of formal effect. Both achieve this replication in the same way, by a qualitative transformation of the medium from transparent to illumined or colored. Color, in short, is subject to the very same physics as light—namely, the physics of multiplication of species. Thus, each spot of illuminated color on the surface of the physical object is presumed to radiate its species outward in all directions through the transparent medium, the species playing formal cause to the medium's material cause.

Struck perpendicularly by this sort of species, the eye's crystalline lens takes on a visible impression of it. The sum total of such impressions at any time gives a complete point-to-point color-representation of the external object as a whole.[41] That representation is called a "visible species." Objectively, the visible species is nothing more than a virtual color-image in the eye of the original object it represents. Like a painting, this image provides necessary clues about the physical attributes of what it represents.[42] Those clues are given through various contrasts and shadings of the colors comprising the visible species. From such clues we gain a subjective impression of the object's shape, size, depth, etc., without actually having those attributes present in us or, for that matter, in the visible species. The visible species thus *intends* the physical nature of its generating object

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not produce sight, which is what impelled him and his followers to assume the necessity of an intervening transparent medium (see *De anima* 2.7.419a12–21; see also *DMS* 1.3, 44 [l. 20]–45 [l. 49]).

[41] The Perspectivists followed Galen in assuming that the crystalline lens of the eye formed the seat of visual sensation. Accordingly, they assumed that incident species impressed their forms upon its surface, by which means they were visually "felt" by the lens. Since every spot on an object radiated its species in all possible directions, each spot on the lens would theoretically receive species from all points on the object's surface. The result would be total confusion. The Perspectivists solved this problem on the basis of the power of perpendicular rays, assuming that the lens was only affected by species arriving along the perpendicular (see my "Getting the Big Picture," 580–83 for details).

[42] Aristotle provides just this analogy in *De memoria et reminiscentia* 1.450b21–26, where he distinguishes between the status of a mural as a picture and a likeness, the one being various daubs of color and the other being a representation that transcends mere color.

without actually possessing it, which is why it is called an "intentional species."[43] Moreover, just as the physical object radiates an intentional species of itself through the transparent medium to the eye, so that intentional species, grasped as a visible species by the eye, radiates an intentional image of itself through the visual spirits of the eye into the brain. This image, called a "sensible species," in turn radiates an intentional image of itself through the animal spirits in the brain, giving rise finally to the "intelligible species." The last in a succession of intentional representations, the intelligible species gives us a full conceptual grasp of the object's essential nature.[44]

We might be tempted to agree with Descartes that this formalist account of vision is unnecessarily complex. If we do, then we can only laud his efforts to deliver us from all those intentional species fluttering through the air.[45] But that is to miss the real point. The entire chain of intentional species and concomitant material causes was carefully designed by the Perspectivists to preserve the essential continuity of the visual process from beginning to end, from object to concept. Thus, since our intellectual grasp of reality is purely formal, its initial cause must be too. In other words, there is a real correspondence between our mental picture of objective reality and the objective reality it pictures. What guarantees this correspondence is the congruence of formal cause and effect at every stage in the visual process as it unfolds from light- and color-radiation, through seeing and perceiving, to apperception. That cause-and-effect congruence assures us of a faithful world-view. Things really are as they appear to be precisely because they come to us by mediation of a continuous series of intentional species, each of which is virtually equivalent to what gives rise to it.[46] No wonder, then, that neither the Perspectivists themselves nor their successors for over three centuries were eager to carry the mechanistic implications of the Perspectivist theory to the literal extreme. To have done so would have meant jettisoning intentional species, which would have meant dispensing with formal causation altogether in favor of material causation. That, however, would have meant destroying the coherence of the visual process by disrupting the cause-and-effect congruence so carefully preserved by the Perspectivist account.

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[43] The intentions carried by intentional species can be understood as implications which we, as animate, attentive beings, infer. When we infer them properly (i.e., with due attention and care), then we intentionally grasp them as they are truly intended (see Sabra, "Sensation and Inference").

[44] On the whole, then the process of vision from its beginning in radiation to its completion in apperception takes place according to a multiplication of intentional species through a continuum of media, from air or water through visual spirits to animal spirits. Each medium has its own "proper object" in the intentional species which it alone is capable of inferring as it is actually intended, so the visual process unfolds at levels of information from mere physical species, which are relatively primitive, to intelligible species, which are highly sophisticated.

[45] See chap. 4, n. 38.

[46] That is, since each new species exists potentially, or by implication, in the previous one, each new species has virtual existence in its predecessor, which has virtual existence in *its* predecessor, and so on.

Against this background, the monumental significance of what Descartes wrought with his wholly mechanistic explanation of light and color becomes abundantly clear. If, as Descartes would have it, light is simply a mechanical impulse or pressure transmitted instantaneously through contiguous particles of Air, then what about color? Either it comes to us by means of another kind of pressure transmitted through the aereal medium, or it is somehow "intended" by light.[47] Forced by logic to choose the latter alternative, Descartes explained color in terms of a spin imparted to the spherical particles of Air when the light-impulse through them is interrupted by crass bodies, such as crystals.[48] Variations in the rate of spin thus give rise to differences in color, red, for instance, being due to a faster spin than blue.[49] It is impossible to overstate the importance of this idea, for, by reducing color to a mechanical by-product or "intention" of light, Descartes took the first and most important step toward eliminating the long-standing distinction between light and color as intimately related but nonetheless independent entities.[50] The process of elimination was completed by Newton, who explained color in terms of varying sizes among light-particles and, on that basis, reduced light (i.e., white light) to a composite of all such particles.[51]

From the standpoint of pure physical explanation, all of this represents a vast improvement over what the Perspectivists had to offer. It is obviously far simpler and more mathematically precise, and, as history shows, it was infinitely more fruitful, yielding insights into hitherto unrecognized and important optical phenomena, such as polarization.[52] As classical mechanics teaches us, however, for every gain there must be a corresponding and equivalent loss. What was lost in this case was all possible correspondence between objective reality and our subjective impression of it. How, after all, can a purely mechanical cause yield a purely formal effect? What possible congruence can there be between our internal impression of red and, say, rate of spin or particle-size? The answer is unequivocal: none whatsoever.

It took time for this point to sink in, but certainly by the end of the seventeenth century it was clear that no effective link could be forged between objective and subjective reality if the former were assumed to be mechanistically determined.[53] Bowing to the inevitable, for instance, John Locke (a devout Newtonian) abdicated all responsibility for establishing such a link:

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[47] In other words, either all visible effects are directly contingent on light-transmission, or they are radiated separately. The simpler alternative is the first, therefore it is the more logical.

[48] Descartes alluded to his theory of spin in the first discourse of the *Dioptrique* (*AT*, 6: 92) but waited until the eighth discourse of his *Météores* to elaborate upon it (*AT*, 6: 331–35).

[49] See *AT*, 6: 334–35.

[50] Ptolemy, *Optics* 2.14–16, 17–18, is undoubtedly the source of this distinction, whence it was perpetuated by the Perspectivists.

[51] Newton's experimental verification that white light is composed of all possible colors can be found in *Opticks* 1, 1, pp. 20–64. Newton was careful to talk about "rays" rather than particles of light, but the inference is obvious: light consists of tiny bodies (see esp. *Opticks* 3, 1, queries 8–11, pp. 340–45). The notion of color-variation by size is clearly expressed in query 13, p. 345: "Do not several sorts of Rays make Vibrations of several bignesses, which according to their bignesses excite Sensations of several Colours, much after the manner that the Vibrations of the Air, according to their several bignesses excite Sensations of several Sounds?" Notice that Newton has reduced the sensation of color to a wave phenomenon.

[52] Newton's idea that rays of light have "several sides," and thus polarity, was adduced to account for double refraction (see *Opticks* 3, 1, Query 26, pp. 358–61).

> I shall not at present meddle with the Physical Consideration of the Mind; or trouble my self to examine, wherein its Essence consists, or by what Motions of our Spirits, or Alterations of our Bodies, we come to have any Sensation by our Organs, or any *Ideas* in our Understandings; and whether those *Ideas* do in their Formation, any, or all of them, depend on Matter or no.[54]

There is in this blunt declaration a certain finality which, though seemingly belied by Locke's temporizing "at present," persisted through his lifetime and, indeed, persists today.[55] The fact remains that we have not yet effectively bridged the gap between pure physical sensations and mental impressions (ideas) which he so clearly acknowledged. Nor do we seem likely to. Why we see "red" or "green," whatever physical account we may provide, is as much a mystery now as it was to Locke at the end of the seventeenth century.

For good or ill, then, the most fundamental and lasting legacy of Cartesian optics was the wholesale and largely unmethodical destruction of the medieval world-view for the sake of a simpler, more mathematically precise account of light and refraction. The resulting mechanistic world-view is no real world-view at all. For, by closing the gap between virtual and real mechanism in the physics of light, Descartes's successors created an unbridgeable chasm between us and the external reality we so desperately strive to understand. Whatever comfort the Scholastics could take from their assurance in a fundamental correspondence between the two is irretrievably lost to us now. But, unlike them, we have the sine-law.

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[53] Descartes's notorious attempt to establish such a link (see *Traité de l'homme* [*AT,* 11: 360–70] and *Passions de l'âme* [*AT* 11: 336–56]) was only one of several; for instance, see Thomas Hobbes's *De homine* of 1658 (in *Opera philosophica,* 5 vols., ed. William Molesworth [London, 1839], vol. 3; a recent account of Hobbes's approach can be found in Tommy L. Lott, "Hobbes's Mechanistic Psychology," in *Thomas Hobbes: His View of Man,* ed. J. G. van der Bend [Amsterdam: Rodopi, 1982). The obvious futility of such reductionist efforts to connect us, as "spiritual" entities, with a mechanistically determined external world made it clear to many that the entire attempt to link the two was a waste of effort.

[54] *Essay Concerning Human Understanding,* ed. Peter H. Nidditch (Oxford: Clarendon Press, 1975), bk. 1, ch. 1, sec. 1, p. 43, ll. 16–22. Locke's abdication of responsibility for explaining any correspondence between ideas and their external causes is wholly consonant with Descartes's position as given in chap. 3, n. 4 above. In bk. 2, ch. 8, sec. 9, p. 135, Locke makes his famous division of perceptible attributes into secondary and primary qualities, the latter supposedly giving rise to simple ideas. Note, however, that his primary qualities ("Solidity, Extension, Figure, Motion, or Rest, and Number") are merely Aristotle's common sensibles, with one Perspectivist visible intention—corporeity (i.e., solidity)—thrown in (see chap. 7, nn. 37 and 39.).

[55] As much as anyone else, Locke was responsible for canonizing the rupture between epistemology and ontology that has been so crucial in the development of modern philosophical and scientific thought.

APPENDICES

Appendix A

The Sine-Law Before Descartes

We are fairly certain that the law of refraction was known to Wilibrord Snel (or Snell) perhaps as early as 1621, and it seems to have been arrived at even earlier, and independently, by Thomas Hariot. In both cases, the law was formulated according to the relation between the refracted ray and the continuation of the incident ray to the image-point. Thus, in trigonometric terms, their version of the law took form as a cosecant-relationship. For a discussion of Snel's work, see J. A. Volgraff, "Snellius' Notes on the Reflections and Refractions of Rays," *Osiris* 1 (1936): 718–25, and Cornelis de Waard, "Le manuscrit perdu de Snellius sur la réfraction," *Janus* 39 (1935): 51–73. For Hariot, see Johannes Lohne, "Thomas Harriott (1560–1621). The Tycho Brahe of Optics," *Centaurus* 6 (1959): 113–21, and John W. Shirley, "An Early Experimental Determination of Snell's Law," *American Journal of Physics* 19 (1951): 507–508. While neither Snel nor Hariot expressed the law according to the sine relationship, Descartes's long-time friend, Claude Mydorge, outlined in 1626 a way of determining the angle of refraction that implies the sine-law (see pp. 27–28 above). He apparently learned of the sine-relationship through Descartes, though (see *Correspondance du P. M. Mersenne,* ed. C. de Waard, et al., 19 vols., [Paris: 1945–83], 1: 404–405). By 1632, certainly, Descartes had formulated the law in a way consonant with his "proof" in the *Dioptrique,* as is evident from his letter to Golius in February of that year, where he suggests a way of determining it experimentally (*AT,* 1: 236–40 and 255 [ll. 25–30]; see also John G. Burke, "Descartes on the Refraction and the Velocity of Light," *American Journal of Physics* 34 [1966]: 390–400). See Schuster, "Descartes and the Scientific Revolution," 308–15, for an excellent and compelling reconstruction of Hariot's and Snel's probable routes to a law of refraction; see also pp. 346–52 for speculation on the date at which Descartes may have arrived at a definitive understanding of the sine-relationship.

Appendix B

The Fermat-Descartes Controversy

Between October of 1637 and April of 1641, Descartes devoted a fair amount of his correspondence to answering objections raised by, among others, Libert Froimond, Pierre de Fermat, Alphonse Pollot, Jean-Baptiste Morin, Pierre Petit, Pierre Bourdin, and Thomas Hobbes (of whom he warned Mersenne exasperatedly "qu'il n'a point du tout de fonds, qu'il a plusieurs opinions extrauagantes, et qu'il tasçhe d'acquerir de la reputation par de mauuais moyens" [Descartes to Mersenne, 4 March 1641, in *AT,* 3: 327 (ll. 1–4)]). Fermat's criticism of Descartes's proof was especially telling, at least in retrospect, because it focused on the mathematical logic rather than the physical or metaphysical grounds of Descartes's argument. Fermat opened the debate with Descartes in a letter to Mersenne (April/May, 1637 [*AT*, 1: 355–61]), but Descartes's response did not come until 5 October of the same year (*AT*, 1: 450–54). Fermat's counter-response came in November or December (*AT*, 1: 464–74), to which Descartes replied in late February or early March of the following year in a letter to Mydorge (*AT*, 2: 15–23). Though evidently dissatisfied with Descartes's defense in this reply, Fermat chose not to answer directly, preferring instead to turn his attention to mathematics. That he refused, nonetheless, to accede to Descartes's reasoning is apparent from a letter of 23 August 1638, in which Descartes grumbled somewhat testily, "ie m'estonne de ce qu'il [i.e., Fermat] continue a vouloir soutenir les obiections qu'il a cy deuant faites contre ma Dioptrique" (Descartes to Mersenne in *AT*, 2: 325 [ll. 20–22]). The actual debate remained closed, however, until 1657—seven years after Descartes's death—when Marin Cureau de La Chambre reopened it (see *OF*, 2: 354). There followed over the next five years a spate of correspondence between Fermat, on the one hand, and Claude Clerselier, Jacques Rohault, and La Chambre (all taking the Cartesian side), on the other (see *OF*, 2: 354–59, 365–74, 382–402, 408–12, 414–29, and 457–84). For a fairly detailed account of this debate, see A. I. Sabra, *Theories of Light*, 116–35, and Michael S. Mahoney, *The Mathematical Career of Pierre de Fermat, 1601-1665* (Princeton: Princeton University Press, 1973), 375–90.

Fermat's counter to Descartes's proof depended on the use of the Principle of Least Time, which Fermat seems to have had in mind from at least 1657 (see *OF*, 2: 354–59). Nonetheless, it was not until January of 1662 that he was able to arrive at what he could accept as a satisfactory method for deducing a law of refraction from that principle (see *OF*, 2: 457–62). The result, he confessed in apparent astonishment, was "le plus extraordinaire, le plus imprévu et le plus heureux qui fût jamais. Car, après avoir couru par toutes les équations, multiplications, antithèses et autres opérations de ma methode, . . . j'ai trouvé que mon principe donnoit justement et precisément la même proportion des réfractions que M. Descartes a établie" (*OF*, 2: 461–62). See pp. 72–74 above for a brief explanation of Fermat's proof. Having published the actual least-time proof of the sine-law in his *Synthesis ad refractiones* of 1662, Fermat was willing to admit that Descartes had shown inspired genius in arriving at the sine-law, but in return he wanted an admission from the other side that the Cartesian "demonstration" was no real demonstration at all. Clerselier's refusal to accept even that compromise drove Fermat to respond tongue-in-cheek that "je lui [i.e., Nature] avois seulement offert un petit secours de géometrie au sujet de la réfraction, si elle en eût eu besoin. Mais puisque vous m'assurez, Monsieur, qu'elle peut faire ses affaires sans cela et qu'elle se contente de la marche que M. Descartes lui a préscrite, je vous abandonne de bon coeur ma prétendue conquête de physique" (*OF*, 2: 483). Unlike Clerselier, Christiaan Huygens (no less a Cartesian than Clerselier) showed a ready willingness to accept the refutation of Descartes's proof: "Car il m'a tousjours semblé, et à beaucoup d'autres avec moy, que mesme Mr. Des Cartes, qui a eu pour but de traitter intelligiblement de tous les sujets de Physique, et qui assurément y a beaucoup mieux reussi que personne devant luy, n'a rien dit qui ne soit plein de difficultez, ou mesme inconcevable, en ce qui est de la Lumiere et de ses proprietez" (*TL*, ch. 1, pp. 6–7).

Appendix C

Kepler, Descartes, and the Anaclastic

The first expression of interest in optical matters that we have from Descartes occurs in the *Cogitationes privatae* of 1619–21 (*AT* 10: 242–43). It was not until the latter part of the 1620s, however, that Descartes evinced a sustained interest in refraction. Precisely what stimulated that interest we will probably never know with any certainty, but we do know that by the second half of the 1620s Descartes had become part of a circle of thinkers concerned with the problem of refraction. Among the members of that circle we can include Isaac Beekman (who played a crucial role in the early development of Descartes's zeal for geometrical reasoning), Jacob Golius, and Claude Mydorge. That circle itself seems to have grown out of an earlier circle that included Thomas Hariot, Willibrord Snel, and Johan Kepler. For a detailed account of both the chronology of Descartes's interest in optics and the contemporary influences (apart from the printed and manuscript sources discussed) that may have stimulated Descartes's interest in refraction, see Schuster, "Descartes and the Scientific Revolution," 300–43.

A major spur to Descartes's interest in refraction was the problem of the anaclastic: the precise curvature of surface that would cause all parallel light-rays striking it to refract into a single focus. Descartes's primary concern with this problem was to perfect optical instruments by perfecting their constituent lenses. To that end he collaborated with a lens-grinder named Ferrier from sometime before mid-1629 to late 1629 in an abortive effort to develop an efficient lens-grinding apparatus (see *AT*, 1: 13–16, 32–69 and 183–87). He was introduced to the problem itself by Kepler (see chap. 2, n. 28 above), who discusses it at length in *Paralipomena*, 4, sec. 5 (*GW* 2: 93–104) and *Dioptrice*, prop. 59 (*GW*, 4: 371–72). In both works, Kepler concluded that the anaclastic was hyperbolic, a conclusion with which Descartes concurred in the eighth discourse of *Dioptrique* (*AT*, 6: 165–96); see also Rule 8 of the *Regulae ad directionem ingenii* (*AT*, 10: 392–400, esp. 393–95).

In his attempt to reconstruct Descartes's real path of reasoning to the sine-law (see chap. 2, n. 11 above), Schuster builds upon the thesis, popularized by Gaston Milhaud (*Descartes savant* [Paris: Librairie Félix Alcan, 1921], 113–18), that Descartes's derivation of the sine-law grew directly out of his efforts to solve the problem of the anaclastic. This effort, in turn, supposedly grew out of his interest in conic sections, an interest that was purely mathematical (cf., however, Sabra, *Theories of Light*, 103, n. 22). By no means implausible, Schuster's reconstruction links Descartes's optical work closely to that of his immediate predecessors, especially Snel and Hariot, whose ideas, Schuster speculates, came to Descartes through Kepler's mediation. By drawing those links, Schuster has certainly shed light on the intellectual network within which Descartes worked. Unfortunately, though, in concentrating as heavily as he does upon that network, Schuster slights the crucial medieval contributions to Descartes's theory.

Appendix D

A Note on the Principle of Natural Economy

Classical ray-theory was predicated on the assumption that the action of light is natural and perfect insofar as it is completed in the shortest possible distance. This assumption underlay Hero of Alexandria's demonstration of the necessity for the Equal Angles Law in reflection through the "Principle of Least Lines." What he proved was that, of all the possible combinations of lines connecting two given points above the surface of a plane or convex mirror to that surface, those two lines that form equal angles with the surface will represent the smallest possible sum. For instance, let ECF in Figure 22 represent the cross-section of a plane or convex mirror, and let A and B represent two given points above that surface. Drop lines from A and B to point C such that angles ACF and BCE are equal. Then drop lines from A and B to some other point D, such that angles ADF and BDE are not equal. Hero's proof consists in demonstrating that AC + BC will always be less than AD

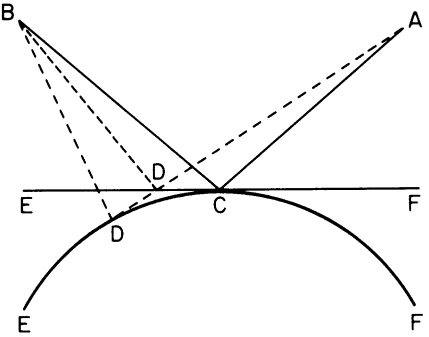

FIGURE 22

+ BD, as long as D and C do not coincide (see *Catoptrics*, props. 4 and 5, pp. 324–31; for a discussion of the foundations of classical ray-theory in the Principle of Least Lines, see my "Saving the Appearances of the Appearances," 86–91 and "Ptolemy's Search," 226–29). Obviously, refraction is not subject to the same governing principle, for if it were, the light would pass straight through without bending since bending lengthens the distance it must traverse to get from one point to another. Therefore, if a rationale for that bending were to be found, it would have to be based upon some principle other than that of least lines or distance.

In casting about for such a rationale, the Perspectivists came quite close to adducing a principle of least action expressed in terms of easiest traversal. A possible source for such a principle is Ptolemy, *Optics* 5.35, 245 (l. 12)–46 (l. 6), where Ptolemy promises to show later in the book how refraction exemplifies "the course of nature in conserving the fulfillments of power":

> Sed si quis opposuerit, dicens qua de causa in primis pretaxatis principiis de perpendicularibus et apparitione rei vidende secundum directionem visibilis radii, coequatur hec species fractionis quam diximus, et fractio que fit a speculis, in quantitatibus autem angulorum non ita fit, quoniam equalitas eorum non permanet hic secundum habitus eius, dinoscetur responsio et quod necessario oportet sic fieri per ea que exposuerimus, ex quibus dinoscetur etiam res mirabilior, videlicet et *cursus naturae in conservandis actibus virtutis* (my emphasis).

Whether and how Ptolemy might have fulfilled this promise is impossible to determine on the basis of the truncated text we now have available. Nevertheless, even without elaboration, the passage just quoted at least suggests a principle of conservation based on dynamic considerations. It is quite possible, therefore, either that Alhazen had the full text available and was simply following Ptolemy's approach in his analysis of the dynamics of refraction or that he was inspired to take that approach by Ptolemy's suggestion alone. Likewise, it is possible that Bacon's more fully developed principle of conservation took its inspiration at least in part from Ptolemy's suggestion.

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